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Comprehensive Exam in Macroeconomic Theory–Procedural Instructions

(1) Write your answers only on the paper we provide.

(2) We are distributing a numbered sign–in sheet in a moment. The number next to your signature will be your **student number**.

(3) Every sheet of paper you turn in to us **must** have your **student number** written at the **top–center** of the sheet and **circled**.

(4) Every sheet of paper you turn in to us **must** have a **page number** written at the **top–right corner** of the sheet.

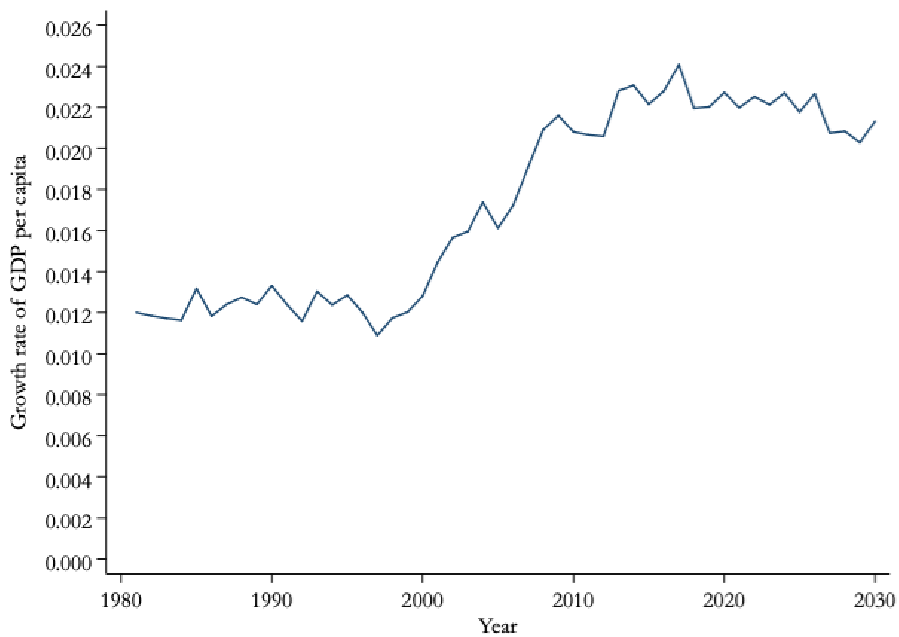
(5) When you have finished, or when it is 5:00 pm (whichever comes first), prepare a cover sheet for your exam. This cover sheet should not have a page number, but must have the following things on it:

(a) Your **student number** at the top–center, circled.

(b) The phrase "Macroeconomics Comprehensive Exam".

(c) The sentence "My last page is page number X", where X is your total number of pages of answers.

1. (20%) You have the following data on the growth rate of GDP per capita. Do not focus on short-run fluctuations in the growth rate, focus on the broad changes.



(a) Write down a model for an economy that includes capital accumulation and savings, *endogenous* technological change and population growth. You are not constrained to any particular type of model. Read the remaining questions first, and based on those write down a model that you feel will help you answer them best. There is no single "right" model.

(b) From your model, derive the steady state growth rate in GDP per capita.

(c) From your model, derive an expression for the steady state capital/output ratio.

(d) Using your model and the expression you derived for (b) and (c), give a plausible explanation for the changes that you see in the figure provided.

(e) Draw a new figure. In that figure, plot the rate of return on capital (R) from 1980 to 2030 implied by your model and consistent with the growth rate of output per capita in the Figure I gave you, and consistent with your explanation in (d).

2. (10%)

(a) (3%) Explain, using a figure, how to find the efficient frontier when a safe asset exists with return r^f. (This is under the assumptions of the CAPM—I ask about the efficient frontier with the safe asset, not the efficient frontier of risky assets although that has to be part of the answer.)

(b) (3%) Write down the formula for the CAPM and explain all terms precisely. (Do not derive the CAPM.)

(c) (4%) Consider two assets. Assume the CAPM holds. Asset A has pay-out PO_A which has a correlation of 0.2 with the market return while asset B has pay-out PO_B which has a correlation of 0.4 with the market return. If the standard deviation of the return to asset A is 1.5 times the standard deviation of the return to asset B, which asset will have the highest expected rate of return?

3. (20%) Consider the case of the 2 agents, Jones (J) and Smith (S), who live for 2 periods in a 3 states-of-the-world economy. Assume each of the agents have utility functions

$$\ln(C_0) + 0.9\, E_0 \ln(C_1) \ .$$

The following table gives the possible endowments and probabilities for Jones and Smith:

	Jones			Smith		
State of the world:	1	2	3	1	2	3
period 0 endowment	50	50	50	50	50	50
period 1 endowment	50	50	50	40	70	50
Probability:	.50	.25	.25	.50	.25	.25

Assume that Jones and Smith are the only two agents in the world.

(a) Assume Jones and Smith in period 0 can trade in a bond that matures in period 1 (equivalently, one can borrow from the other in period 0) but not in any other assets. What is the rate of interest? (If you state the two equations in two unknowns that determines the solution *and* states whether the interest rate is positive, 0, or negative, that is considered a full answer.)

(b) Now assume that J and S can trade in Arrow securities for state 1 and state 2 (there are no Arrow securities for state 3). Find the prices of the Arrow securities.

(c) Under the assumption of perfect Arrow-Debreu markets (full set of Arrow securities), find the rate of interest between period 0 and period 1. (For this question, it is sufficient to state the equations that would need to be solved, rather than finding the explicit solution.)

(d) Explain (without solving, but using concepts from class) whether the interest rate is higher or lower in case (c) than in case (a).

4. (30%) Consider the infinite horizon problem of a risk averse individual that wants to experiment in the labor market and is deciding when to have a child. Experimentation in the labor market can bring higher future earnings, but it is risky and thus costly (since he/she is risk averse). At the same time, the agent faces another trade-off since a kid brings many pleasures but it is also costly in terms of the labor career. Thus, the timing of the childbirth is important as it affects the possibility of experimentation in the beginning of the career which lead to higher life-time earnings.

In the beginning of the period, the agent has assets a, a known skill x, and may or may not have a child. The agent can only have one child. If the agent starts the period without a child, an 'unplanned' childbirth may occur with probability ρ. Each period, after the exogenous childbirth shock is realized, the agent needs to decide whether to have a child if there is no child yet. Children are costly but generate additional utility. The agent's instantaneous utility depends on consumption and whether or not there is a child; denote by $u_0(c)$ the utility of a childess individual and by $u_1(c)$ the utility of a parent. There is a fixed cost ϕ of having a child, and there may also be a variable cost κ (the rate at which the earnings are reduced).

In addition, the agent decides whether to experiment with a new skill. Experimenting may result in drawing a higher skill (e.g., finding a better occupational match). Formally, if the agent experiments, her new productivity y is drawn from $F(y)$, and the next period's productivity is

$$x' = \max\{(1-\delta)x, (1+\gamma)y\}.$$

Here δ is the rate at which the unused skill depreciates, and γ is the rate at which the used skill grows. If the agent does not experiment, the next period's skill is

$$x' = (1+\gamma)x.$$

Denote by w the wage rate per unit of skill, resulting in earnings wx (or wy) for the agent that operates skills x (or y) in the current period. Finally, the agent decides how much to consume and save in a risk-free asset; the risk-free interest rate is r. The time discount rate is β.

(a) Consider the individual with no child who wants to continue his/her life without a child but wants to pursue his/her existing skills, i.e does not want to experiment in the labor market. Denote his/her value function as $V_{0S}(\cdot)$. Formulate the dynamic programming problem for this individual (i.e. formulate the optimization problem by writing the Bellman equation). Similarly, consider the individual with no child but who wants to experiment in the labor market. Denote the value function for such an individual by $V_{0R}(\cdot)$. Formulate the dynamic programming problem for this individual.

Note: You are required to figure out the state and control variable(s) as well as to write down the problem recursively but you do not need to solve the problem as it does not have closed form solution. Please specify the variables and be clear with the notation.

(b)Now consider the individual that wants to have a child and wants to pursue his/her existing skill. Denote the value function for such an individual by $V_{1S}(\cdot)$. Formulate the dynamic programming problem for this individual. Finally, consider the value of having a child but for an individual who wants to experiment in the labor market and denote her/his value function by $V_{1R}(\cdot)$. Formulate the dynamic programming problem for this individual.

Note: You are required to figure out the state and control variable(s) as well as to write down the problem recursively but you do not need to solve the problem as it does not have closed form solution. Please specify the variables and be clear with the notation.

(c) What is the value of having no child in a particular period? What is the value of having a child in a particular period? Provide the expression of the value functions and briefly provide your interpretation.

(d) The probability ρ can be interpreted as the technology for controlling the timing of childbirth, i.e. higher (lower) ρ means less (more) control over childbirth. Suppose that technological improvements make ρ lower. How can this affect earnings over life-cycle? How can this affect higher standard deviation of earnings in the cross-section?. Here we just ask you to briefly provide your intuition.

5. (20%) There are a large number (measure N) of identical households. We will assume a complete set of contingent Arrow-Debreu securities that are traded across households. Each household has one unit of time, which it allocates between labor and leisure. The househoold also accumulates capital, and rents the capital to firms. Firms hire labor and rent capital to produce output and maximize profits. The production function is Cobb-Douglas with capital share α. Workers take wages as given and firms take prices as given. Capital depreciates at rate δ. The source of uncertainty is an aggregate economy-wide productivity shock:

$$ln(A_t) = \rho ln(A_{t-1}) + \epsilon_t \tag{1}$$

where ϵ_t has a normal distribution with mean 0 and variance σ^2. Importantly, households and firms make decisions in period t *after* they observe the productivity shock in period t.

Preferences are given by:

$$E\left[\sum_{t=0}^{\infty}\beta^t u(c_t, l_t)\right] \tag{2}$$

where c_t is household consumption, and l_t is household labor, in period t, and:

$$u(c_t, l_t) = \frac{1}{1-\gamma}\left(c_t - \psi\frac{l_t^{1+\theta}}{1+\theta}\right)^{1-\gamma} \tag{3}$$

For your answers below use small letters for household variables and capital letters for economy-wide variables. Make sure you clearly identify all variables.

(a) (2) Write out the household budget constraint.

(b) (4) Set up the LaGrangian problem of the households and derive the first order conditions.

(c) (3) Look at the labor-leisure first order condition. Is there a wealth effect (i.e., an effect from changes in permanent income) on labor supply? Why or why not, i.e., explain?

(d) (1) State the goods market equilibrium condition.

(e) (7) Now set up the **value function** form of the social planner's problem. Make sure it is clear what the state variables are and what the control variables are. Make sure the "subject to ..." equations are clear, too.

(f) (3) Verify that the social planner's labor-leisure choice corresponds to what you derived in part b above.