Preliminary Exam Material
Applicable Analysis

Topics:

Contraction mappings and fixed points. Applications of contractions mappings: integral equations, solutions to initial value problems. Local existence and uniqueness of solutions, stability.

L^p spaces as metric completions. Extending the Riemann integral to L^p spaces. Banach spaces.

Dual spaces. Uniform boundedness.

Consequences of uniform boundedness for Fourier series and polynomial interpolation.

Uniform convexity, best approximation property and duality for L^p-spaces.

Bounded inverse, closed graph theorem.

Hilbert spaces. Orthonormal bases and their characterization. Characterization of best approximation by orthogonal projection.

Fourier series. Convergence in L^2 and pointwise convergence. Weak convergence.

Relationships between weak and norm convergence. Weak compactness in Hilbert spaces. Linear programming in Hilbert spaces.

Operators and bilinear forms. The Lax-Milgram theorem.

The Hilbert-Schmidt norm and Hilbert-Schmidt operators. Compact self-adjoint operators. The spectral theorem for compact, self-adjoint operators. Diagonalizing normal operators.

Separation theorems (separating hyperplanes). Hahn-Banach theorem. Consequences of separation for support vector machines.

Text: Part of the material (Contraction Mappings, Fourier series) is covered in Chs. 9, 11, and 14, K. Davidson and A. Donsig, Real Analysis with Applications: Theory in Practice, Springer, 2014. The main portion of material on Functional Analysis is in John Hunter and Bruno Nachtergaele, Applied Analysis, available online at https://www.math.ucdavis.edu/∼hunter/book/pdfbook.html