Syllabus: University of Houston PhD Qualifying Examination – Probability

(1) Probability triples

(2) Probabilistic foundations
- (a) Random variables
- (b) Independence
- (c) Continuity of probabilities
- (d) Borel-Cantelli lemma
- (e) Tail fields
- (f) Kolmogorov zero-one law

(3) Expected values (integration)

(4) Distributions of random variables
- (a) Discrete distributions
- (b) Absolutely continuous distributions
- (c) General case (Borel measure on $\mathbb{R}$ induced by a random variable)

(5) Inequalities
- (a) Markov inequality
- (b) Chebyshev inequality
- (c) Cauchy/Schwarz inequality
- (d) Jensen inequality

(6) Convergence of sequences of random variables
- (a) Pointwise convergence almost everywhere
- (b) Convergence in probability
- (c) Convergence in distribution (weak convergence)

(7) Integration-to-limit theorems
- (a) Monotone convergence theorem
- (b) Lebesgue dominated convergence theorem
- (c) Fatou lemma

(8) Probabilistic limit theorems
- (a) Strong law of large numbers
- (b) Central limit theorem

(9) Convolution, Fubini/Tonelli theorem

(10) Conditioning
- (a) Conditioning on a random variable or a sub-σ-algebra
- (b) Conditional expectation, conditional probability
- (c) Law of total probability

(11) Discrete-time, discrete-space Markov chains
- (a) Building transition matrices
- (b) First-step analysis
- (c) Branching processes
- (d) Irreducible Markov chains, reducible Markov chains, communication classes
- (e) State types: Positive recurrent, null recurrent, transient
- (f) Period of a state, aperiodic Markov chain
- (g) Stationary distributions: Fundamental results on existence, uniqueness, and convergence

(12) Poisson processes
- (a) Poisson process, nonhomogeneous processes
- (b) Distributions associated with the Poisson process
- (c) Link between the uniform distribution and Poisson processes
- (d) Spatial Poisson processes
- (e) Compound Poisson processes, marked Poisson processes