

ECONOMICS 7330 – Probability and Statistics, Fall 2024

Homework 6. Due Wednesday October 9.

1. Assume that X and Y follow a normal distributions with means μ_x and μ_y . Denote the covariance between X and Y σ_{XY} and the variance of X σ_X^2 and similarly for the other variances covariances.
 - a) Write down the joint density of X and Y using scalars.
 - b) Find the conditional density $F(X|Y)$ by dividing the density from part a) with the marginal density of Y .
 - c) Write down mean and variance of X, Y in vector/matrix form (the variance matrix is 2 by 2, for example). Write down the density in vector-matrix notation. (You have the formulas for the multivariate Normal, so be explicit what goes into the matrices.)
2. Assume $X \sim N(0, 9)$, $Y \sim N(2, 9)$, and $Z \sim N(2, 16)$. Further assume that the covariance between X and Y is 2, while both X and Y are independent of Z .
 - i) What is $E(X|Y = 2, Z = 3)$? (State the formula you use and then the number.)
 - ii) What is the conditional variance $Var(X|Z = 3)$?
 - iii) What is $E(X, Z|Y = 3)$?
3. Consider an i.i.d. sample $X_1, \dots, X_N$. Define the residual $e_i = X_i - \bar{X}$. Verify that $\bar{X}e_i = 0$.
4. Consider two i.i.d. samples $X_1, \dots, X_N$ and $Y_1, \dots, Y_N$ with X_i correlated with Y_i . Find the expected value of $\sum_{i=1}^N (X_i - \bar{X})(Y_i - \bar{Y})$.