Go over syllabus.

No laboratory meeting this week.

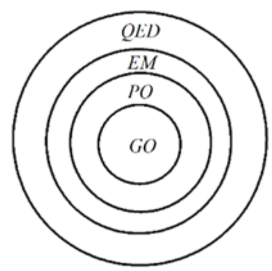
Why study optics?

1. Technological implications – CDs, bar codes, optical integrated circuits, etc.
2. Theoretical importance – beam shaping, fast light, slow light, interaction with matter, etc.
3. Applications to other areas – materials, optometry, metrology, etc.

Traditionally, optics is taught by first introducing geometrical optics and then proceeding to the electromagnetic theory version and physical optics. This approach makes sense because we first need to understand how to get light where we want it and then proceed to the more complicated issues of diffraction, interference, polarization, and Fourier optics.

Geometrical optics deals with lenses, mirror, prisms, etc.
Wave optics or physical optics deals primarily with diffraction and interference.
The electromagnetic approach adds polarization, i.e., the vector nature of light.
Quantum electrodynamics (QED) deals with electron-photon interactions.

Each successive version includes the earlier treatments as special cases, and I will try to illustrate that to the extent it is possible to do so at this level. Here is the basic idea in figure form to the right.



One approach to geometrical optics is based on Fermat's principle of extremal time. One can predict how light travels in space, in media, and at boundaries, but one cannot predict how much light goes where using geometrical optics. In other words, we cannot predict how much light is transmitted or reflected at a boundary – that requires other approaches. The law of refraction (Snell's law) and the law of reflection follow from Fermat's principle of extremal time. Fermat's principle was first stated as the principle of least time, but it is not hard to show that some optical paths are maximum, whereas some are neither.

Fermat's original statement: The actual path between two points taken by a beam of light is the one that requires the least time. The modern statement is that the optical path light takes in traveling from point S to point P is an extremal. That is, it can be a maximum, minimum, or inflection point. We will see examples of each of these in due time.

The optical path length (OPL) is defined by the equation

$$OPL \equiv \int_S^P n(s)ds,$$

where s is the arc length along the path. This equation looks deceptively simple, but it is quite difficult to apply in general cases. The problem is that we are usually trying to find the OPL, but in order to find it we need to know $n(s)$. We have a vicious circle.

In general, $t = \frac{OPL}{c}$, and for homogeneous media, where n is not a function of s, OPL = nS. We define the index of refraction as $n = \frac{c}{v}$, so that $t = \frac{nL}{c} = \frac{L}{v}$.

From these ideas, we will see shortly how Snell's law and the law of reflection follow. Geometrical optics then follows from these two laws and the small angle approximations of them. The advantage is simplicity, but the disadvantages include a failure to include diffraction, interference, and polarization effects, lack of information about the amount of light transmitted and reflected, and some algebraically complicated calculations.

The physical optics or wave approach is based on Huygen's principle, which states that every point on a wavefront acts as a new source of spherical wavelets such that their superposition creates the wavefront at any time in the future. Of course, we know that waves do not reradiate, electrons do, but the model works, so it is used in spite of its incorrect assumptions.

Fresnel later modified this principle to calculate the new wave by using a superposition of the wavelets to include amplitude and phase (Huygens – Fresnel principle).

Huygens principle accounts for the gross features of diffraction, but not the structure in the pattern. The Huygens – Fresnel principle would later become the Fresnel – Kirchhoff scalar diffraction theory. This theory gives a good account of diffraction and interference with quantitative results, but it does not account for polarization, *i.e.*, the vector nature of light.

The electromagnetic approach is based on Maxwell's equations, which give rise to a wave equation, thus showing the validity of the superposition principle. Maxwell's equations are given by

$$\nabla \cdot \mathbf{D} = \rho; \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}; \quad \nabla \cdot \mathbf{B} = 0; \quad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}.$$

The so-called constitutive equations are given by

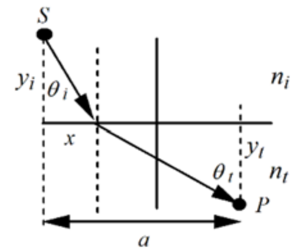
$$\mathbf{D} = \epsilon_o \epsilon_r \mathbf{E}, \quad \mathbf{B} = \mu_o \mu_r\, \mathbf{H}, \quad \text{and } \mathbf{J} = \sigma \mathbf{E}.$$

The Lorentz force equation given by $\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$ is the final piece of information from which all classical effects are calculated. Later, we will see that the electric field is the primary light vector, as it dominates in almost all materials with the exception of highly magnetic materials. In free space and integral form, Maxwell's equations are given by

$$\oint E \cdot dS = \frac{q}{\epsilon_o}, \qquad \oint B \cdot dS = 0, \quad \oint E \cdot d\ell = -\frac{d\Phi_B}{dt}, \quad \oint B \cdot d\ell = \mu_o i + \mu_o \epsilon_o \frac{d\Phi_E}{dt}.$$

Quantum electrodynamics is a relativistic quantum mechanical of the electromagnetic field. A probability P is associated with each event with $P \propto |A|^2$.

Now, let's use Fermat's principle to derive Snell's law (law of refraction). Here is a figure that represents the problem. A ray of light starts from S and travels to P. Our objective is to determine the value for x such that the path is extremal. Note that we have already assumed that both rays are in the same plane and that each medium is homogeneous. The electromagnetic treatment confirms that this assumption is valid.



$$OPL = n_i(x^2 + y_i^2)^{1/2} + n_t((a - x)^2 + y_t^2)^{1/2}$$

so

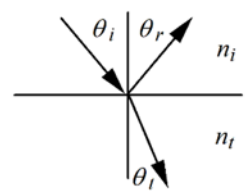
$$\frac{d(OPL)}{dx} = \frac{n_i\left(\frac{1}{2}\right)(2x)}{(x^2 + y_i^2)^{1/2}} + \frac{n_t\left(\frac{1}{2}\right)2(a - x)(-1)}{((a - x)^2 + y_t^2)^{1/2}}$$

$$= n_i \sin\theta_i - n_t \sin\theta_t = 0 \implies n_i \sin\theta_i = n_t \sin\theta_t.$$

The incident ray, normal to the surface, and the refracted (transmitted) ray all lie in the same plane. Note that we use the subscript *t* for the refracted ray to avoid confusion when we use *r* for the reflected ray. It is worth noting that the relationship between θ_i and θ_t is not linear but involves the sines of the angles. Notice, furthermore, that our derivation does not require the ray to travel from *S* to *P*. Light could just as easily travel from *P* to *S*, and the results would be the same. This leads us to the principle of optical reversibility. Fermat's principle was a precursor to Hamilton's principle of least action in mechanics. In fact, in more advanced texts, you may find a section on Hamiltonian optics. Fermat's principle is also related to the calculus of variations that you might recall from mechanics when you derived Lagrange's equations of motion.

The law of reflection is derived in a similar way, but I will only show the results here. Here, the angles are all independent of the indices of refraction in each medium. They all lie in the same plane, and the law of reflection becomes

$$\theta_i = \theta_r.$$



Suppose you place a point source at one focal point in an elliptical mirror. Where would you expect the llight to be focused? Is the OPL a maximum, minimum, or neither? Think about how you were taught to make an ellipse to answer these questions.

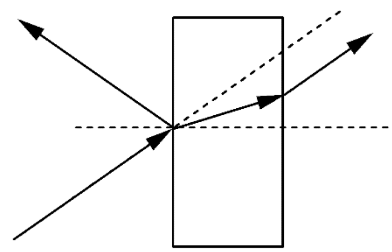
Let's think of the angle of incidence as the input and the angle of refraction as the output and solve Snell's law for the angle of refraction, which we call the output of the optical system. We obtain

$$\theta_t = \sin^{-1}\left[\frac{n_i}{n_t}\sin\theta_i\right].$$

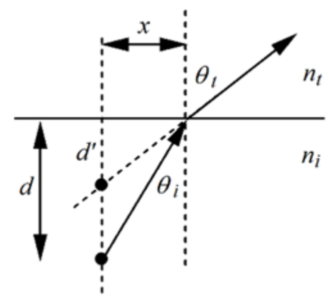
This expression clearly shows the relationship between θ_i and θ_t to be nonlinear. In many applications, lenses in particular, we must linearize this expression to obtain simple expressions concerning the lenses. That means we use $n_i\theta_i \cong n_t\theta_t$. Under what conditions may we use this approximation?

Some important applications of Snell's law:

Windows and beam splitters- Windows are used in cases where we need to isolate a region. These regions are typically high temperature, low temperature, high pressure, or low pressure (vacuum). Beam splitters are used when we need to take a single beam of light and divide it into two or more beams. Interferometers are one common example of this use. Here is a figure that shows what happens to a beam when it strikes a window or beam splitter. Are these the only beams that result from the interaction? Explain.



Apparent depth – The figure shows a point a distance *d* below the surface of water. The ray that an observer above the water sees appears to have come from a distance *d*' below the water. Our objective is to relate *d* and *d*'. Here $n_t < n_i$, so $\theta_t > \theta_i$ as shown. Note that $x = d\tan\theta_i$ and $x = d'\tan\theta_t$. Therefore,



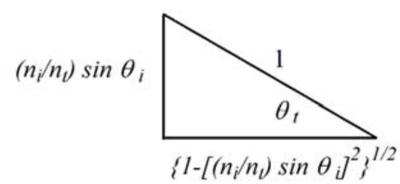
$$d\tan\theta_i = d'\tan\theta_t$$

so

$$\frac{d'}{d} = \frac{\tan\theta_i}{\tan\theta_t}.$$

Now use Snell's law $n_i\sin\theta_i = n_t\sin\theta_t$ and rewrite as

$\theta_t = \sin^{-1}\left[\frac{n_i}{n_t}\sin\theta_i\right]$ and use $\sin\theta_t = \frac{n_i}{n_t}\sin\theta_i$ along with the triangle shown in the figure. It is possible to express everything is terms of the refractive indices and the angle of incidence. Therefore, we express



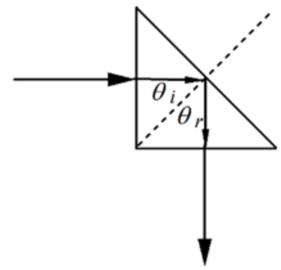
$$\tan\theta_t = \left[\frac{n_i}{n_t}\sin\theta_i\right]\left[1 - \left(\frac{n_i}{n_t}\sin\theta_i\right)^2\right]^{-1/2}.$$

We see that the relationship between *d* and *d'* is highly nonlinear. An extended object is likely to show some distortion because of this nonlinearity. If, however, all angle are small enough that we can make the approximation $\theta \cong \tan\theta \cong \sin\theta$, then $x = d\theta_i, x = d\theta_t$, and $n_i\theta_i \cong n_t\theta_t$. These equations are said to be linearized, and, as you can see, are much easier to work with. Now you can see that $d' = \frac{n_t}{n_i}d$, so for a tank of water, $d' = \frac{1}{4/3}d = \frac{3}{4}d$.

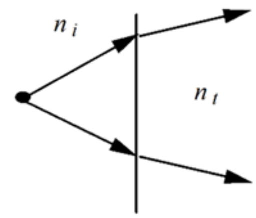
Total internal reflection – The figure above shows that whenever light travel from a medium of higher refractive index to one of lower refractive index, the angle of refraction is larger than the angle of incidence. This means that θ_t will reach 90 degrees for some angle of incidence less than 90 degrees. For this angle and angle larger than this, the refracted ray is parallel to the surface, and the beam is totally reflected. Here is how the mathematics works.

$$n_i \sin\theta_i = n_t \sin\theta_t \;\; and \;\; \sin\theta_i = \frac{n_t}{n_i}\sin\theta_t$$

Whenever $\theta_t = \frac{\pi}{2}, \theta_i = \theta_c$, so $\theta_c = \sin^{-1}\frac{n_t}{n_i}$. θ_c is said to be the critical angle for total internal reflection (TIR). This process is the most efficient method for reflecting light and is the basis for the operation of some prisms. Here is a figure that shows how a 45º – 45º – 90º prism reflects light. Assuming that glass has a refractive index of 1.5, the critical angle is 41.8º, so the angle of incidence of 45º causes TIR. Optical instruments use prisms for the excellent reflective properties.



Notice that light that transmits through a flat surface does not change from convergin to diverging or from diverging to converging, so flat surfaces have no imaging properties. Consider, however, what would happen if he refractive index could be made negative. Over the past 15 years or so, it has become possible to make such materials (manmade) so flat surfaces can be used as imaging devices. These materials are known as metamaterials.



NEXT TIME: Curved surfaces, lenses, and imaging using refraction and transfer matrices.