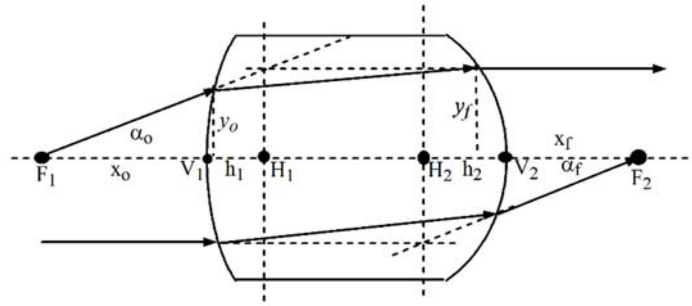
LAST TIME: Fiber optic review, general treatment of thick lenses and lens systems, principal points, and examples

Recall

$V_1H_1 = h_1$ $F_1H_1 = f_1$ $V_2H_2 = h_2$
$F_2H_2 = f_2$ $x_o = F_1V_1$ $x_f = F_2V_2$.

Therefore, $f_1 = x_o + h_1$ and $f_2 = x_f + h_2$.

Finally, $f_1 = -\frac{a_{11}a_{22}-a_{21}a_{12}}{a_{12}} = -\frac{1}{a_{12}}$



$$h_1 = f_1 - x_o = -\frac{1}{a_{12}} - \frac{y_o}{\alpha_o} = -\frac{1}{a_{12}} - -\frac{a_{11}}{a_{12}} = \frac{1-a_{11}}{-a_{12}}.$$

Similarly,

$$h_2 = \frac{a_{22}-1}{-a_{12}}$$

We recover the thin lens equation as long as distance for s_o, s_i, and f are measured with respect to the principal points.

What does all this mean?

We have established that the points H_1 and H_2 are points where refraction can be thought of to bo concentrated. We have also establishe that the distances h_1 and h_2 are simply related to the system matrix. We still do not have a simply relationship between s_o, s_i, and f. Let's see what happens when so and si are measured with respect to H_1 and H_2. We will trace a ray from an object to its image and use the imaging condition to determine the relationship. I will write the matrix operations first and then explain what is going on.

It is also possible to find H_1 and H_2 for each element in the system and apply the thin lens equation *n* times. It is also possible to apply the object-image matrix without regard to H_1 and H_2, but in this case, object and image distances are measured from V_{11} and V_{2n}.

For a single thick lens the result are given by

$$h_1 = -\frac{f(n_l-1)d_l}{R_2 n_l}, h_2 = -\frac{f(n_l-1)d_l}{R_1 n_l} \text{ and } \frac{1}{f} = (n_l-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n_l-1)d_l}{n_l R_1 R_2}\right].$$

Here is another example using the system matrix and principal points. Rather than just give you all of the parameters, I will give you the system matrix and some of the parameters.

Example

The system matrix for a thick bi-convex lens in air is given by

$\begin{bmatrix} 0.6 & -2.6 \\ 0.2 & 0.8 \end{bmatrix}$. (a) If $R_1 = 0.5$ cm, $d_l = 0.3$ cm and $n_l = 3/2$, calculate R_2, and the focal length, and locate the principal points. (b) For an object placed 1.0 cm from V_1, where is the image located with respect to V_2? (c) Characterize completely the image.

It would be wise to check that this system matrix is actually possible. How do we do that?

The usual approach is to first determine the focal length. We do this by recalling that

$$f = -\frac{1}{a_{12}} = -\frac{1}{-\frac{13}{5}} = \frac{5}{13} \text{ cm.}$$

Then, we use the equation for the focal length of a single thick lens given by

$$\frac{1}{f} = (n_l - 1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n_l - 1)d_l}{n_l R_1 R_2}\right].$$

Before we even start the calculation, what do you expect the sign of R_2 to be and why?
We substitute numbers to obtain

$$\frac{13}{5} = \left(\frac{3}{2} - 1\right)\left[\frac{1}{0.5} - \frac{1}{R_2} + \frac{\left(\frac{3}{2} - 1\right)0.3}{\left(\frac{3}{2}\right)0.5R_2}\right].$$

Simplifying this equation gives

$$\frac{26}{5} = \left[2 - \frac{1}{R_2} + \frac{1}{5R_2}\right] \Rightarrow \frac{1}{5R_2} - \frac{5}{5R_2} = \frac{16}{5} \Rightarrow R_2 = -\frac{1}{4} \text{ cm.}$$

Use

$$h_1 = -\frac{f(n_l - 1)d_l}{R_2 n_l} = -\frac{5}{13}\frac{\left(\frac{3}{2} - 1\right)0.3}{\left(-\frac{1}{4}\right)\frac{3}{2}} = \frac{2}{13} \text{ cm}$$

and

$$h_2 = -\frac{f(n_l - 1)d_l}{R_1 n_l} = -\frac{5}{13}\frac{\left(\frac{3}{2} - 1\right)0.3}{\left(\frac{1}{2}\right)\frac{3}{2}} = -\frac{1}{13} \text{ cm}$$

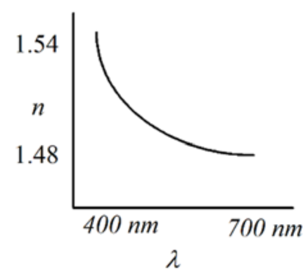
Calculate $s_o = (2/13 + 13/13) = 15/13$ cm and $\frac{1}{s_i} = \frac{1}{f_o} - \frac{1}{s_o} = \frac{13}{5} - \frac{13}{15} = \frac{26}{15} \Rightarrow s_i = \frac{15}{26}$ cm.

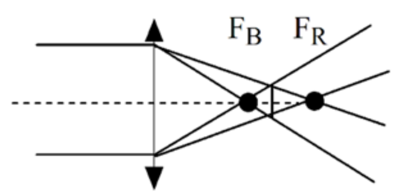
Because $h_2 = -\frac{1}{13}$ and is to the left of V_2, we need to subtract it from s_i to get the image position with respect to V_2. Therefore, our result is given by $\frac{15}{26} - \frac{2}{26} = \frac{1}{2}$ cm. The magnification is then given by $M_T = -\frac{s_i}{s_o} = -\frac{\frac{1}{2}}{\frac{15}{13}} = -\frac{13}{30}$, so image is reduced and inverted. I have put some extra example problems on the web site for your practice.

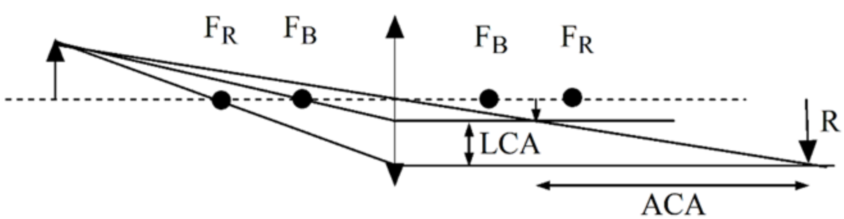
Aberrations fall into two general categories, chromatic and monochromatic. Chromatic aberrations arise from the dependence of the index of refraction on the wavelength. Because the focal length of a lens depends on the index of refraction, object and image positions are also dependent on the wavelength. This dependence is called dispersion, and is usually written as

$$n = n(\lambda) \Rightarrow \frac{1}{f(\lambda)} = (n_l(\lambda) - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{s_o} + \frac{1}{s_i(\lambda)} = \frac{1}{f(\lambda)}.$$

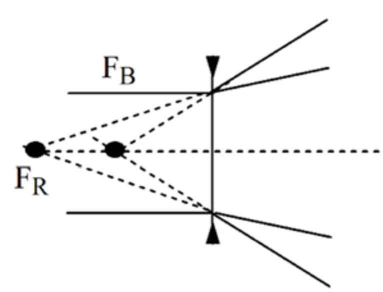
This is the mathematical statement that a single object will form multiple images if the light incident on a lens is polychromatic – many colors or wavelengths. Recall that for normal dispersion, the refractive index varies with wavelength according to the following figure. For a given lens, the focal length of the blue light is slightly smaller than that of the red light. Therefore, the effect of the lens on white light is somewhat like the figure shown. The distance between the red and blue focal points is called the axial chromatic aberration, and the vertical line between the two focal points is called the circle of least confusion. The figure below is a ray trace that shows how the image is affected by the chromatic aberration.



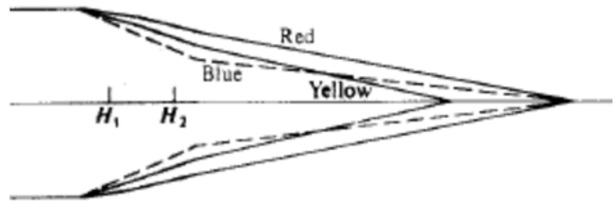




How do we correct for chromatic aberration? Let's look at the behavior of a concave lens with chromatic aberration. This gives us a hint that we might use a concave lens in combination with a convex lens to reduce the chromatic aberration. Here is how the idea works. You may think of a lens as having a prism effect because it is either thicker or thinner at the middle that it is at the edges. We might think of combining two thin lenses to help minimize the effect of chromatic aberration. Here is a figure from your textbook showing how the idea works. See the next page. If we put two think lenses together, the effective focal length is given by



$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$. We write the focal length for each lens at the wavelength of yellow light. Why do you suppose a spceific wavelength of yellow is chosen?



$$\frac{1}{f_{1y}} = (n_{1y} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)_1$$

and

$$\frac{1}{f_{2y}} = (n_{2y} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)_2$$

We will glue the lenses together so that $d = 0$ in the expression above. Therefore,

$$\frac{1}{f_y} = \frac{1}{f_{1y}} + \frac{1}{f_{2y}} = (n_{1y} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)_1 + (n_{2y} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)_2.$$

The right-hand side of the expression above is just the total refractive power D of the two lenses glued together. If the refractive power does not change with wavelength, then the chromatic aberration shoud disappear. We calculate the derivative of D with respect to λ and set the result equal to zero as follows.

$$\frac{\partial D}{\partial \lambda} = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)_1 \left(\frac{\partial n_1}{\partial \lambda}\right)_y + \left(\frac{1}{R_1} - \frac{1}{R_2}\right)_2 \left(\frac{\partial n_2}{\partial \lambda}\right)_y = 0.$$

We must approximate the derivative with $\frac{\Delta n}{\Delta \lambda}$ and evaluating the result between red and blue. This means that

$$\frac{\partial D}{\partial \lambda} \cong \left(\frac{1}{R_1} - \frac{1}{R_2}\right)_1 \left[\frac{n_{1R} - n_{1B}}{\lambda_R - \lambda_B}\right] + \left(\frac{1}{R_1} - \frac{1}{R_2}\right)_2 \left[\frac{n_{2R} - n_{2B}}{\lambda_R - \lambda_B}\right].$$

Now recall that $D_1 = (n_{1y} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)_1$ so we may rewrite the expression above as

$$\frac{D_{1y}}{(n_{1y} - 1)}\left[\frac{n_{1R} - n_{1B}}{\lambda_R - \lambda_B}\right] + \frac{D_{2y}}{(n_{2y} - 1)}\left[\frac{n_{2R} - n_{2B}}{\lambda_R - \lambda_B}\right] = 0.$$

Define the dispersive power V to be given by

$$V = \frac{n_y - 1}{\lambda_R - \lambda_B} = \text{Abbe V number}.$$

Finally,
$f_{1y}V_1 + f_{2y}V_2 = 0$ is the condition for $f_R = f_B$. Lenses that have been created in this way are know as acromatic doublets. The lines that are typically used come from the naming of the various emission lines in gases. Here are some typical values used.

d is the yellow helium line at 587.5 nm, F is the blue hydrogen line, and C is the red hydrogen line at 656.2 nm. Your textbook lists several of the strong Fraunhofer lines for your information. Optical glasses have a bit of jargon used to name them, but it is worth knowing so here it is. The wavelength chosen is usually the center of the sodium doublet, d5esignated as the sodium D line, which is at 589.29 nm. The parameters are given as (n_D - 1):(10V_D). If you were given the parameters as 517:645, you would have a glass with $n_D = 1.517$ and $V_D = 64.5$. High V numbers have a lower dispersive power. Your textbook has a table that showd many glasses that are commonly used.

Monochromatic aberrations (von Seidel)

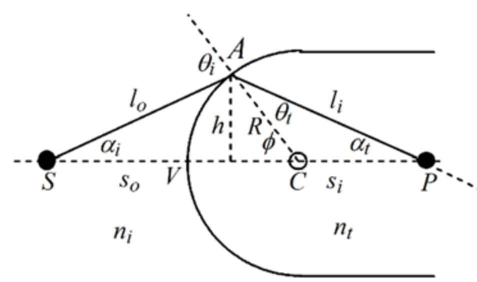
Image degrading (blurred images)

Spherical aberration
Coma
Astigmatism

Image deforming (clear, but not a replica of the object)

Field curvature
Distortion

I will not spend an enormous amount of time on aberrations, but you should at least be aware of them and what causes them. Some of them are not difficult to correct. To develop the ideas, we recall how we started the refraction process and the approximations we had to make at the time. Here is the figure we have used earlier. We define the aberration function $a(A)$ as follows.



$a(A) = (n_i l_o + n_t l_i) - (n_i s_o + n_t s_i)$. We have already calculated the values of l_o and l_i. Now, however, we will use $\sin\phi = \frac{h}{R}$ and $\cos\phi = (1-\sin^2\phi)^{1/2} \cong 1 - \frac{h^2}{2R^2} - \frac{h^4}{8R^4}$.
With some algebra, we obtain

$$l_o = s_o\left(1 + \left[\frac{h^2(R+s_o)}{Rs_o^2} + \frac{h^4(R+s_o)}{4R^3 s_o^2}\right]\right)^{1/2}$$

and

$$l_i = s_i\left(1 + \left[\frac{h^2(R-s_i)}{Rs_i^2} + \frac{h^4(R-s_o)}{4R^3 s_i^2}\right]\right)^{1/2}$$

Now use

$$(1+x)^{1/2} \cong 1 + \frac{x}{2} - \frac{x^2}{8}$$

to get

$$l_o = s_o\left(1 + \left[\frac{h^2(R+s_o)}{2Rs_o^2} + \frac{h^4(R+s_o)}{8R^3 s_o^2} - \frac{h^4(R+s_o)^2}{8R^2 s_o^4}\right]\right)$$

and

$$l_i = s_i\left(1 + \left[\frac{h^2(R - s_i)}{2Rs_i^2} + \frac{h^4(R - s_o)}{8R^3 s_i^2} - \frac{h^4(R - s_o)^2}{8R^2 s_i^4}\right]\right)$$

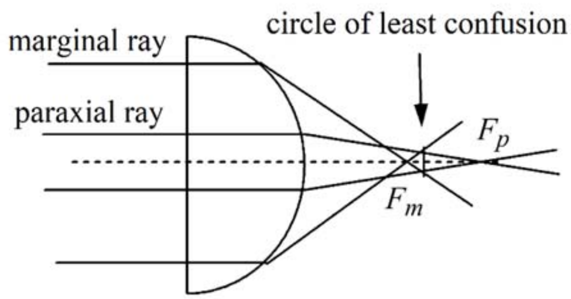
If we group our h^2 and h^4 terms together, we get, for the h^2 terms, the same equation we got before for the refraction at a single surface. However, for the h^4, we obtain

$$a(A) = -\frac{h^4}{8}\left[\frac{n_i}{s_o}\left(\frac{1}{s_o} + \frac{1}{R}\right)^2 + \frac{n_t}{s_i}\left(\frac{1}{s_i} - \frac{1}{R}\right)^2\right]$$

In other words,

$$a(A) = ch^4.$$

Here is a figure showing spherical aberration.



NEXT TIME: Other aberrations and waves