LAST TIME: Linear polarizers, general treatment of anisotropic materials, uniaxial crystals, wave plates, Jones vectors, and Jones matrices

$$\frac{OPD}{\lambda} = \frac{\pi/2}{2\pi} = \frac{1}{4} \text{ so } OPD = \frac{\lambda}{4} = |n_f - n_s|d.$$

$$\epsilon = \frac{2\pi}{\lambda} OPD$$

$$d = \frac{\lambda_o}{4|n_e - n_o|}$$

Jones vector

$$\begin{bmatrix} E_{ox} \\ E_{oy} e^{i\epsilon} \end{bmatrix}$$

LP states: $E_{ox}\ or\ E_{oy} = 0;\ \ \epsilon = \pm 2n\pi;\ \ \epsilon = \pm(2n+1)\pi$

CP states: $\epsilon = +\frac{\pi}{2};\ \ E_{ox} = E_{oy} = E_o\quad LCP\ ccw$
$\epsilon = -\frac{\pi}{2};\ \ E_{ox} = E_{oy} = E_o\quad RCP\ cw$

EP states: $\epsilon = +\frac{\pi}{2};\ \ E_{ox} \neq E_{oy} \neq 0\quad LEP\ ccw$ axes ∥ coordinate axes.
$\epsilon = -\frac{\pi}{2};\ \ E_{ox} \neq E_{oy} \neq 0\quad REP\ cw$ axes ∥ coordinate axes
ϵ arbitrary, $E_{ox} \& E_{oy} \neq 0$; elliptical with axes not parallel to coordinate axes

Usually Jones vectors are normalized, so if we wanted to write a linearly polarized wave at 45 degrees, we would write

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

In general, a normalized LP state at an arbitrary angle θ with respect to the x-axis is written as

$$\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

The list of Jones matrices is given in your textbook, but because it is useful to have as we work through problems, here they are.

Horizontal linear polarizer: $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, vertical linear polarizer: $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Linear polarizer at 45°: $\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, linear polarizer at -45°: $\frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

QWP – fast axis vertical: $e^{i\pi/4}\begin{bmatrix}1 & 0\\ 0 & -i\end{bmatrix}$, QWP – fast axis horizontal: $e^{i\pi/4}\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$

RCPolarizer: $\frac{1}{2}\begin{bmatrix}1 & i\\ -i & 1\end{bmatrix}$, LCPolarizer: $\frac{1}{2}\begin{bmatrix}1 & -i\\ i & 1\end{bmatrix}$

Let a horizontal LP act on an arbitrary LP state to obtain

$$\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix} = \begin{bmatrix}\cos\theta\\ 0\end{bmatrix}$$

To get circularly polarized light,

$$e^{i\pi/4}\begin{bmatrix}1 & 0\\ 0 & -i\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \frac{e^{i\pi/4}}{\sqrt{2}}\begin{bmatrix}1\\ -i\end{bmatrix} = \frac{e^{i\pi/4}}{\sqrt{2}}\begin{bmatrix}1\\ e^{-i\pi/2}\end{bmatrix}$$

Here, $\epsilon = -\pi/2$, so we have RCP (cw rotation). The QWP has its fast axis vertical, so the y-component travels faster than the x-component. If we now let our arbitrary LP state be acted on by a QWP with its fast axis horizontal, we obtain

$$e^{i\pi/4}\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix} = e^{i\pi/4}\begin{bmatrix}\cos\theta\\ i\sin\theta\end{bmatrix}.$$

To make this a bit easier to understand, let's pick a value for θ, say 30 degrees. Then, our equation for the outpur becomes

$$\frac{e^{i\pi/4}}{2}\begin{bmatrix}\sqrt{3}\\ i\end{bmatrix} = \frac{e^{i\pi/4}}{2}\begin{bmatrix}\sqrt{3}\\ e^{i\pi/2}\end{bmatrix} = e^{i\pi/4}\begin{bmatrix}\sqrt{3}/2\\ e^{i\pi/2}/2\end{bmatrix}.$$

From these results, we see that $E_{ox} = \frac{\sqrt{3}}{2}, E_{oy} = \frac{1}{2}$, and $\epsilon = +\frac{\pi}{2}$. Therefore, this is left elliptically polarized light. According to our previous results, the axes of the ellipse should be aligned with the coordinate axes. We may check using the equation

$$\tan 2\alpha = \frac{2E_{ox}E_{oy}\cos\epsilon}{E_{ox}^2 - E_{oy}^2} = 0 \text{ because } \cos\frac{\pi}{2} = 0.$$

Let's look at the results of the video I showed where we reflected circularly polarized light from a mirror and found that wave did not get back through the QWP and linear polarizer. Here is how the Jones vector and matrices would look. We start with the LP state and go thorough the QWP.

$$e^{i\pi/4}\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \frac{e^{i\pi/4}}{\sqrt{2}}\begin{bmatrix}1\\ i\end{bmatrix}.$$

Now let the mirror act to obtain

$$\frac{e^{i\pi/4}}{\sqrt{2}}\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}1\\ i\end{bmatrix} = \frac{e^{i\pi/4}}{\sqrt{2}}\begin{bmatrix}-1\\ -i\end{bmatrix} = -\frac{e^{i\pi/4}}{\sqrt{2}}\begin{bmatrix}1\\ i\end{bmatrix}.$$

$$-e^{i\pi/4}\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}\frac{e^{i\pi/4}}{\sqrt{2}}\begin{bmatrix}1\\ i\end{bmatrix} = -\frac{e^{\frac{i\pi}{2}}}{\sqrt{2}}\begin{bmatrix}1\\ -1\end{bmatrix}.$$

Finally, back through the linear polarizer to obtain

$$-\frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\frac{e^{\frac{i\pi}{2}}}{\sqrt{2}}\begin{bmatrix}1\\ -1\end{bmatrix} = -\frac{e^{\frac{i\pi}{2}}}{2\sqrt{2}}\begin{bmatrix}0\\ 0\end{bmatrix}.$$

So, as we saw, no light gets back through the system, but it is because the electric field vector is rotated 90 degrees when the light is reversed and travels through the QWP. Note that to see the phase of the field as it re-enters the linear polarizer. You can see this because $-1 = e^{i\pi}$. To extend this discussion a bit, consider what happens when linearly polarized light passes through two identical QWPs.

$$e^{i\pi/4}\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}e^{i\pi/4}\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \frac{e^{i\pi/2}}{\sqrt{2}}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix} = \begin{bmatrix}1\\ -1\end{bmatrix}.$$

This state is orthogonal to the original state and so would not pass through the same linear polarizer. The device that is represented by the Jones matrix

$$e^{i\pi/2}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$

is a half wave plate.

Just to be sure we are understanding more complicated cases, consider the following. Suppose we write a Jones vector in what we might call a Cartesian form and want to determine how to write it in a polar form. Consider

$$\begin{bmatrix}A\\ B+iC\end{bmatrix} = \begin{bmatrix}A\\ \left(\sqrt{B^2+C^2}\right)e^{i\epsilon}\end{bmatrix} = \begin{bmatrix}A\\ \left(\sqrt{B^2+C^2}\right)e^{i\tan^{-1}(C/B)}\end{bmatrix}.$$

Therefore,

$$E_{ox} = A;\ E_{oy} = \sqrt{B^2+C^2}; \text{and } \epsilon = \tan^{-1}(C/B)$$

Now consider

$$\begin{bmatrix}3\\ 1+2i\end{bmatrix} = \begin{bmatrix}1\\ 1\end{bmatrix} + 2\begin{bmatrix}1\\ i\end{bmatrix} = \begin{bmatrix}1\\ 1\end{bmatrix} + 2\begin{bmatrix}1\\ e^{i\pi/2}\end{bmatrix}.$$

One way to think of this state is the superposition of an LP state at 45 degrees plus an LCP state. This state, however, should be written as a single state, and we can see what that is when we go back to the original form and write is in the standard form. Therefore, using the result above, we obtain $A = 3 = E_{ox}, E_{oy} = \sqrt{B^2+C^2} = \sqrt{1^2+2^2} = \sqrt{5}$, and $\epsilon = \tan^{-1}(C/B) = \tan^{-1} 2$.

Finally,

$$\tan 2\alpha = \frac{2E_{ox}E_{oy}\cos\epsilon}{E_{ox}^2 - E_{oy}^2} = \frac{2(3)\left(\sqrt{5}\right)\cos[\tan^{-1}(2)]}{9-5} = \frac{6\sqrt{5}}{4}\frac{1}{\sqrt{5}} = \frac{3}{2}$$

and

$$2\alpha = \tan\frac{3}{2} = 56^o \text{ and } \alpha \cong 28^o.$$

To go further, we could go back to our original equations and make some plots to see exactly the handedness and the form of the ellipse, but I leave that to you and to an example on the web site.

Videos from MIT Optics

NEXT TME: Finish polarization and start interference – Chapter 9