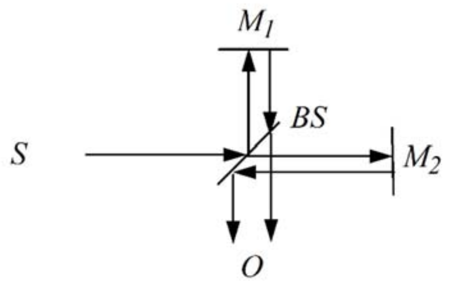
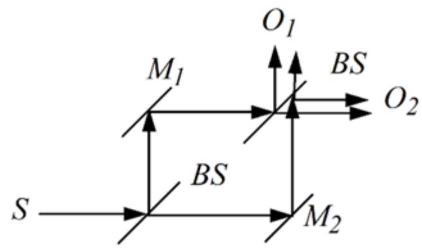
LAST TIME: Interferometers and multiple-wave interference

Interferometers are extremely useful devices for measuring small differences in quantities. The two most common two-wave interferometers are the Michelson interferometer and the Mach-Zehnder interferometer. Here are figures showing how each one of them functions.

Michelson interferometer



Mach-Zehnder interferometer



Multiple wave interference by amplitude division – see next page.

r = external reflection; r' = internal reflection; t = external transmission; t' = internal transmission
Recall internal and external reflections and define internal and external transmissions.

$$E_{or} = E_o r + E_o t'r'te^{-i\delta} + E_o r'^3 tt'e^{-2i\delta} + \cdots + E_o tt'r'^{(2N-3)}e^{-i\delta(N-1)}.$$

and

$$E_{or} = E_o\left\{r + r'tt'e^{-i\delta}\left[1 + r'^2e^{-i\delta} + \left(r'^2e^{-i\delta}\right)^2 + \cdots + \left(r'^2e^{-i\delta}\right)^{(N-2)}\right]\right\}$$

The term in the square brackets is a sum that has the form $1 + x + x^2 + \cdots$, which is recognized as $\frac{1}{1-x}$.
Therefore, we obtain

$$E_{or} = E_o\left[r + \frac{r'tt'e^{-i\delta}}{1 - r'^2e^{-i\delta}}\right].$$

With some algebra and trigonometry substitutes, we obtain

$$I_r = I_i\frac{\left[\frac{2r}{(1-r^2)}\right]^2\sin^2\frac{\delta}{2}}{1 + \left[\frac{2r}{(1-r^2)}\right]^2\sin^2\frac{\delta}{2}} \text{ and } I_t = I_i\frac{1}{1 + \left[\frac{2r}{(1-r^2)}\right]^2\sin^2\frac{\delta}{2}}.$$

As we would expect $I_r + I_t = I_i$ if no absorption is present. Let's examine the maxima and minima of these equations.

$$(I_t)_{max} = I_i \text{ when } \delta = 2m\pi \text{ and } (I_r)_{min} = 0$$

If, however, $\delta = (2m+1)\pi$ so that $\frac{\delta}{2} = (2m+1)\frac{\pi}{2}$

$$(I_t)_{min} = I_i \frac{(1-r^2)^2}{(1+r^2)^2} \text{ and } (I_r)_{max} = I_i \frac{4r^2}{(1+r^2)^2}$$

Define $F = \left(\frac{2r}{1-r^2}\right)^2$ = coefficient of finesse and write

$$\frac{I_r}{I_i} = \frac{F\sin^2\frac{\delta}{2}}{1+F\sin^2\frac{\delta}{2}};\ \frac{I_t}{I_i} = \frac{1}{1+F\sin^2\frac{\delta}{2}};\ \frac{1}{1+F\sin^2\frac{\delta}{2}} = \text{Airy function}$$

Sometimes, you will write the finesse as $\mathcal{F} = \frac{\pi}{2}\sqrt{F}$. But much of this is just renaming things to agree with historical naming of terms.

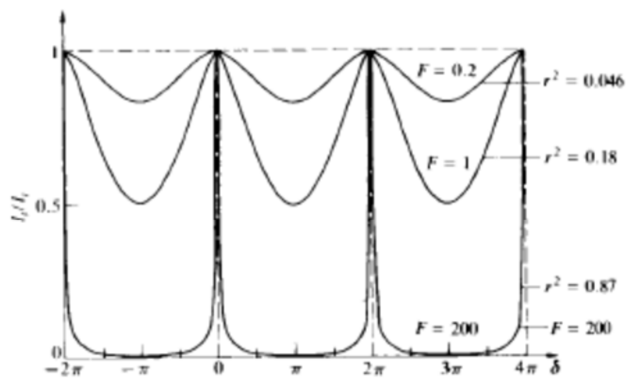
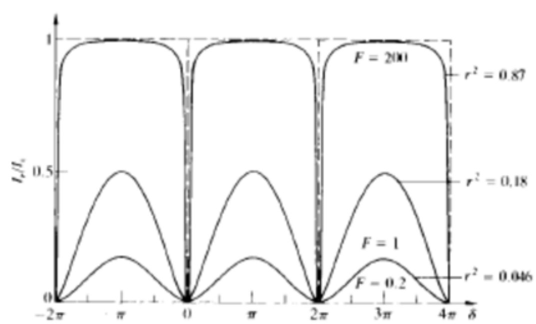


Figure 9.41 Airy function.



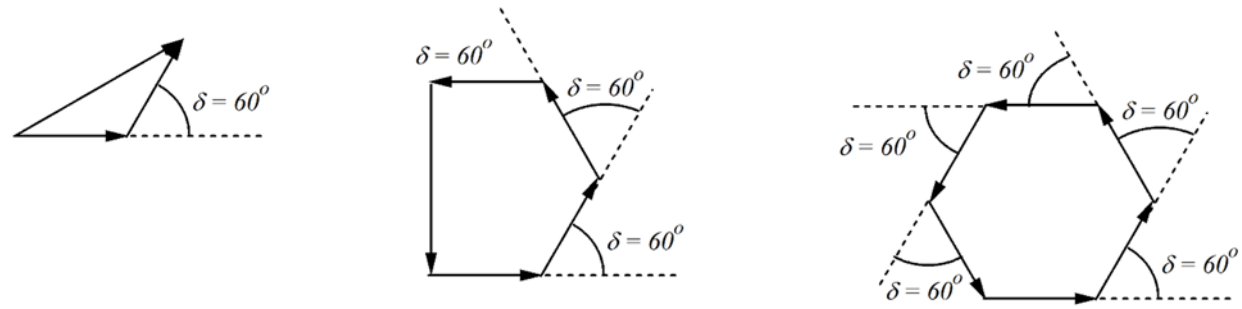
igure 9.42 One minus the Airy function.

Videos 18 and 19.

How do we try to understand the difference between the sharpness of the fringes for two-wave and multiple-wave interference?

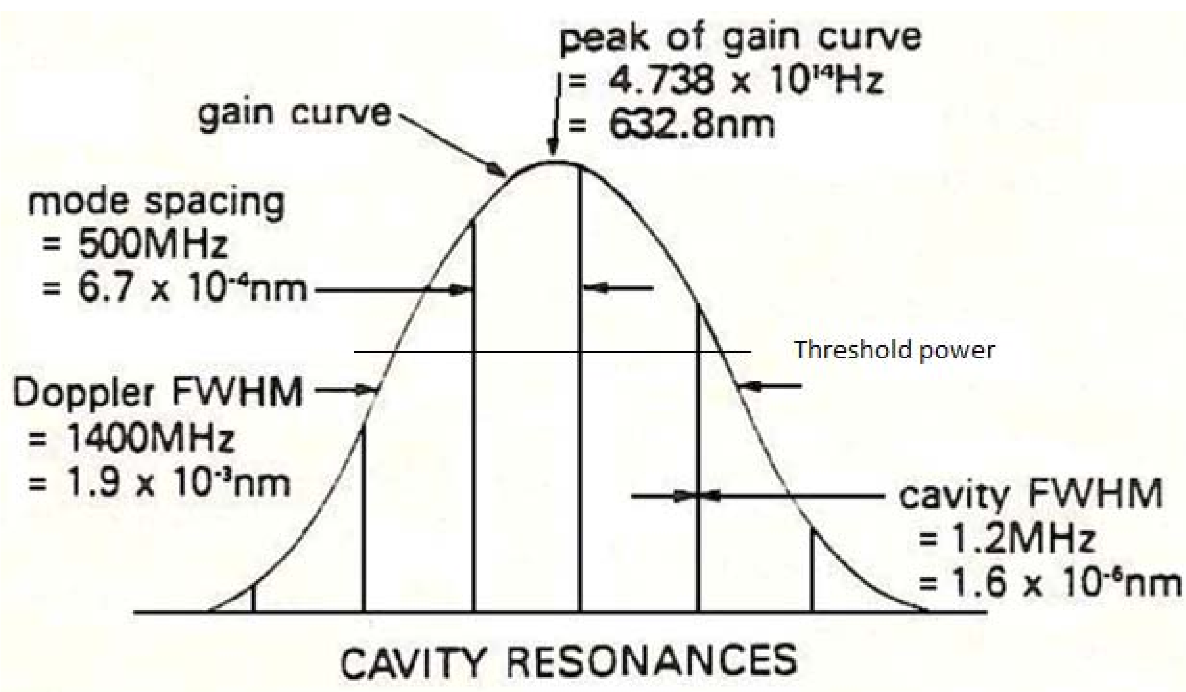
The easiest way to visualize the difference is to go back and look at the concept of phasors and how they add together for two waves and multiple waves. Let's suppose that we can create a phase shift of 60 degrees between two waves and between six waves, all having equal amplitudes. On the following page, I show the phasor addition for the two cases, drawn to scale.

Two-wave phasor addition Four-wave phasor addition Six-wave phasor addition



What you see from here is that with only two waves adding together, you must have a phase difference of 180 degrees in order to get to zero. With four waves, you would require a phase difference of 90 degrees to get to zero, and with six waves, you only require 60 degrees of phase difference. If you had 120 waves adding together, you would need only 3 degrees of phase difference to get a zero sum. Therefore, the line width of the interference gets smaller the more interfering waves you have. This is the reason that the Fabry – Perot cavity has such a narrow line width. The higher the reflectivity of each mirror, the more reflections we have and the more waves that add together.

We will digress a bit to see how some of what we have studied applies to the optics of the laser. Describe the basics of the laser and show the following curve.



As I have drawn this laser, it would have 3 different frequencies operating. Because the end of the laser cavity are almost perfect reflectors, it has some similarities to the vibrating string that you studied in your introductory course. Specifically, you must have an integer number of half wavelengths inside the cavity. Therefore, $n\frac{\lambda}{2} = L$, but $\lambda\nu = c$ so $\frac{2L}{n}\nu_n = c$ or $\nu_n = \frac{nc}{2L}$.

This means that the spacing of the frequencies is given by

$$\Delta\nu = \frac{c}{2L} = \frac{3 \times 10^8}{0.6} = 5 \times 10^8 \text{ Hz or } 500 \text{ MHz.}$$