Cornu Spiral 1

Let $z$ extend from -∞ to +∞. The width along the $y$ axis is 1.5 mm. Let $\rho_o = 0.5$ m, $r_o = 1.0$ m, and $\lambda = 500$ nm. Therefore, $z_1 = -\infty$, $z_2 = +\infty$, $v_1 = -\infty$, $v_2 = +\infty$, $y_1 = -0.75$ mm, and $y_2 = +0.75$ mm. Now we calculate $u_1$ and $u_2$, so

$$u_1 = -0.75 \times 10^{-3} \left[\frac{2(1.50}{500 \times 10^{-9} \times 0.5 \times 1.0}\right]^{\frac{1}{2}} = -2.6.$$

By symmetry, $u_2 = +2.6$ and $\Delta u = 5.2$ Substitute these values into the equation for the irradiance at $P$ to obtain

$$I_P = \left(\frac{I_o}{4}\right)\{[2\mathcal{C}(2.6)]^2 + [2\mathcal{S}(2.6)]^2\}\{[2\mathcal{C}(\infty)]^2 + [2\mathcal{S}(\infty)]^2\}$$
$$= \left(\frac{I_o}{2}\right)\{0.605 + 1.213\} = 0.91 I_o.$$

Cornu Spiral 2

Change the previous problem to a plane wave incident on the aperture. Therefore, $\rho_o$ becomes infinite, and our value for $u$ is given by

$$u = y\left[\frac{2}{\lambda r_o}\right]^{1/2} \Rightarrow u_1 = -1.5;\ \ u_2 = +1.5;\ \ \Delta u = 3.0$$

Now

$$I_P = \left(\frac{I_o}{2}\right)\{[2\mathcal{C}(1.5)]^2 + [2\mathcal{S}(1.5)]^2\} = 1.37 I_o.$$

Cornu Spiral 3

We will keep the same aperture size with a plane wave incident and determine what value of $\lambda$ gives the greatest irradiance. Now the use of the Cornu spiral becomes very important. We can see from the Cornu spiral that the longest chord occurs when $u_1 \cong -1.2$ and $u_2 \cong +1.2$. Now we use the same expression above, but we are looking for $\lambda$. Therefore,

$$1.2 = 0.75 \times 10^{-3}\left[\frac{2}{\lambda(1.0)}\right]^{1/2} \Rightarrow \lambda = 781 \text{ nm}.$$

Now we may substitute these values back into the equation for $I_P$ and obtain $I_P = 1.78 I_o$. It is not hard to show that this wavelength corresponds to a situation where the diameter of the first Fresnel zone is equal to the width of the slit. If the slit length were changed to give a square aperture, we find that the irradiance is just the square of 1.78, so $I_P = 3.16 I_o$. Finally, if we changes the aperture to a circle to match the one zone, we get $I_P = 4 I_o$ as before.

Cornu Spiral 4

How do we obtain off-axis information concerning irradiance? As we stated before, we move the aperture to keep the relative position of the observation point the same with respect to the edges of the aperture. Therefore, still using the same geometry as before, let's move point P up by 0.45 mm. We consider this equivalent to moving the aperture down by 0.45 mm. Now, the upper edge of the aperture $y_2$ = 0.3 mm and the lower edge of the aperture $y_1$ = -1.2 mm. The values for $u_1$ and $u_2$ are $u_1 = -2.4$ and $u_2 = 0.6$. Note that $\Delta u = 3.0$.

$$I = \frac{I_o}{2}\{[\mathcal{C}(0.6) - \mathcal{C}(-2.4)]^2 + [\mathcal{S}(0.6) - \mathcal{S}(-2.4)]^2\} = 0.912 I_o.$$

Cornu Spiral 5

Now let's move all the way to where the upper edge of the aperture is at the geometric shadow. In other words, $y_2$ = 0 and $y_1$ = -1.5 mm. Then $u_2 = 0$ and $u_1 = -3.0$.

Cornu Spiral 6

How do we get a minimum in the center of the pattern? Once again, the Cornu spiral comes to the rescue. We can see that the minimum chord length occurs when $u_2 = 1.9$ and $u_1 = -1.9$.

$$\Delta u = 3.8$$

Moving off the axis once again here implies that we could use $u_2 = 2.3$ and $u_1 = -1.5$. Then we see that $I = 1.32\ I_o$

Cornu Spiral 7

Half plane from Hecht

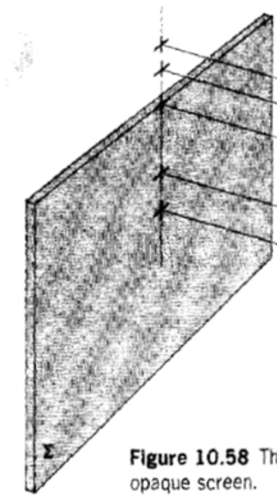
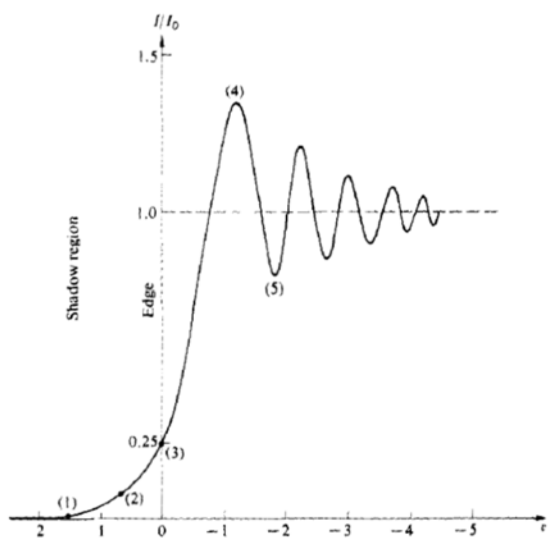



**Figure 10.58** The semi-infinite opaque screen.



Cornu Spiral 8

Barrier is essentially two apertures.  Add the two components as vectors to account for the phase.