# THE NEXUS OF ROLLOVER AND INTEREST-RATE RISKS IN SOVEREIGN DEFAULT MODELS *

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**Abstract**

This paper develops a sovereign default model in which rollover risk generates interest-rate multiplicity. The mechanism relies on a complementarity between slow- and fast-moving debt crises: pessimistic beliefs about future rollover risk raise interest rates today, increasing debt service and validating the initial pessimism. The model generates volatile spreads and rich debt dynamics even without fundamental shocks. Calibrated to Spain, it captures key features of the European debt crisis and the effects of interventions by the European Central Bank. With fundamental shocks, calibrations to Spain and Mexico show that this complementarity is amplified, becoming quantitatively more relevant.

**Keywords**: Sovereign default, self-fulfilling crises, multiple equilibria

**JEL Classification Numbers: E44, F34**

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# 1 Introduction

A central question in the study of sovereign debt crises is whether creditor coordination failures are quantitatively important. In this paper, we develop a sovereign debt model in which self-fulfilling crises arise through a novel mechanism: the complementarity between rollover risk, as in Cole and Kehoe (2000), and interest-rate risk, as in Calvo (1988). A higher perceived probability of a future rollover crisis raises interest rates today, which in turn increases debt service obligations and raises the likelihood of such a crisis, thereby validating the higher interest rates through a self-fulfilling mechanism. Our quantitative analysis shows that the model accounts for the spreads and debt dynamics observed in Spain during the European sovereign debt crisis and that this complementarity is essential to match the level and volatility of spreads, as well as debt-service levels, in the data for both Spain and Mexico.

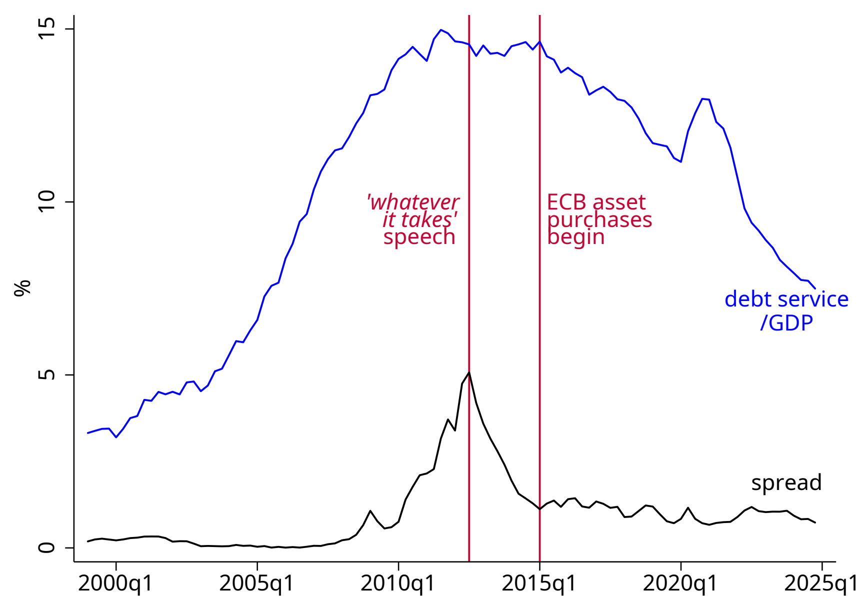



Figure 1: Debt service and interest rate spreads in Spain (2000–2025)

Spain's experience during the European sovereign debt crisis, illustrated in Figure 1, highlights the potential role of self-fulfilling crises. In July 2012, the President of the European Central Bank (ECB), Mario Draghi, attributed the widening in sovereign spreads to a crisis of confidence among creditors rather than economic fundamentals.[1] This interpretation shaped the ECB's response: first through Draghi's pledge to do "*whatever it takes*" to preserve the euro, and shortly thereafter through the announcement of the Outright

[1]The full speech is available at: https://www.ecb.europa.eu/press/key/date/2012/html/sp120726.en.html.

Monetary Transactions (OMT) program, which authorized the ECB to purchase sovereign bonds in secondary markets without ex ante quantitative limits.[2] Sovereign spreads declined sharply after these announcements, even though the ECB never conducted any purchases under the OMT program.

By integrating the slow-moving dynamics of interest rates and debt from Calvo (1988) with the fast-moving rollover crises of Cole and Kehoe (2000) into a unified framework, the model captures both the buildup of debt and spreads and the subsequent dynamics observed during the Spanish debt crisis. In this framework, a self-fulfilling increase in interest rates can push the government into a rollover crisis region, leaving it vulnerable to shifts in market sentiment even in the absence of shocks to fundamentals. We show that while credible policy announcements by a lender of last resort can eliminate high spreads, direct interventions such as bond purchases or bailouts are required for the government to exit the rollover crisis region. This result is consistent with the sequence of events observed during the crisis: policy announcements by the ECB in the summer of 2012 sharply reduced spreads but had limited effects on debt-to-GDP ratios; only after the launch of its new bond-buying program in March 2015 did debt-to-GDP ratios and debt service begin to decline.

Rollover risk, as in Cole and Kehoe (2000), arises when countries rely on new borrowing to finance current debt service. If creditors expect an imminent default on previously issued debt, the price of newly issued bonds falls, preventing the country from raising sufficient funds to repay maturing obligations and potentially triggering a default. Models of rollover risk have been used to explain sharp reversals in capital inflows at the height of debt crises, such as the Mexican crisis of December 1994 or the European crisis in the summer of 2012.[3] A key assumption in these models concerns the timing of actions: the borrower issues new debt before deciding whether to default on previously issued debt.

In contrast, interest-rate risk, as in Calvo (1988), arises because higher interest rates can themselves raise debt service and push countries closer to default. Models with this type of interest-rate multiplicity have recently gained renewed attention as an explanation for episodes in which sovereign spreads rise gradually over time, such as in Europe after

[2]On August 2, 2012, the ECB announced it would take action in secondary sovereign bond markets, later defined on September 6, 2012 as "Outright Monetary Transactions" with its details: `https://www.ecb.europa.eu/press/pr/date/2012/html/pr120906_1.en.html`.

[3]This source of multiplicity was explored in Cole and Kehoe (1996), Conesa and Kehoe (2017), Bocola and Dovis (2019), Aguiar et al. (2022), and Bianchi and Mondragon (2022).

2008.[4] A key assumption in these models concerns the actions available to the borrower. We assume that the borrower chooses the amount of revenue to raise from bond markets, leaving the amount to be repaid in the future determined by the market.[5] High interest rates make default more likely, which in turn validates the high interest rates.

Our model incorporates both mechanisms by adopting the timing assumption in Cole and Kehoe (2000) and the borrower action assumption in Calvo (1988). By demonstrating the complementarity between the two mechanisms, we depart from existing literature that considers these modeling assumptions in isolation. We begin by illustrating our core mechanism in a simple three-period model where the borrower issues Calvo-type debt in the first period and faces a Cole–Kehoe-style rollover risk in the second period. Without any income fluctuations, we show analytically that this setup can produce interest-rate multiplicity that is generated by the rollover risk. If creditors expect a rollover crisis in the second period, they will charge a higher interest rate on the debt issued in the first period. But this larger debt service in the second period may in turn push the borrower to default in case a rollover crisis occurs, hence justifying the higher initial interest rate.

Guided by the results of our stylized three-period model, we extend the analysis to an infinite-horizon setup to test the complementarity between the two sources of multiplicity. We maintain the assumption of no income shocks; hence, the only source of risk comes from two sunspot variables that exogenously drive the market sentiment regarding rollover and interest-rate risk. Calibrated to Spain, the model generates rich debt and bond spread dynamics driven by the interaction between these two forms of multiplicity. Each simulated path eventually enters a slow-moving debt crisis, in which debt accumulation is fueled by creditor coordination on high interest rates (a bad Calvo sunspot). As debt and interest rates rise, the borrower enters a crisis zone and becomes vulnerable to a rollover crisis (a bad Cole–Kehoe sunspot), which results in a default. While a temporary and unexpected policy announcement by a lender of last resort can bring the spread down to zero, it does not, by itself, reduce debt levels or lead to an exit from the crisis zone. A lasting solution requires actual intervention that delivers sufficient debt relief.

Finally, we augment the model with income growth shocks estimated using GDP growth data for Spain, as well as for Mexico, a common emerging market benchmark in the lit-

[4] This source of multiplicity was explored in Aguiar and Amador (2020), Lorenzoni and Werning (2019), and Ayres et al. (2018, 2025).

[5] An alternative is to assume the borrower chooses the amount to be repaid in the future, as in Arellano (2008), which leads to equilibrium uniqueness. See Ayres et al. (2018) and Lorenzoni and Werning (2019).

erature that features a starkly different set of targeted moments. The results show that the complementarity between the rollover and interest-rate risks generates the level of spreads and their volatility, as well as the debt-service levels observed in the data. We show that the model captures a wide range of combinations of average debt service-to-GDP ratio, as well as the level and volatility of interest rate spreads. Importantly, the richer dynamics in our model arise primarily from the interaction between the two sources of multiplicity, rather than from simply stacking them together. Thus, our model also demonstrates that a straightforward combination of the two mechanisms of self-fulfilling crises in the literature can create synergy in replicating the empirical moments for debt and spreads that quantitative models typically aim to match.

## 1.1 Literature review

This paper is closely related to the sovereign default literature with self-fulfilling debt crises. The two foundational papers for our approach are Calvo (1988) and Cole and Kehoe (2000). Our contribution is to show that a complementarity exists between the two mechanisms, and that it is quantitatively significant. In a closely related paper, Corsetti and Maeng (2024) also study a model with both types of multiplicity to show that the two types of debt crises can arise in a unified framework for different state variables. By contrast, our paper highlights a complementarity between the two modeling assumptions and studies the quantitative implications of their interaction, in particular in a framework with no additional sources of uncertainty.

Our results are also related to recent papers that incorporate equilibrium multiplicity in the spirit of Calvo such as Lorenzoni and Werning (2019), Ayres et al. (2018, 2025), Stangebye (2020) and Bianchi and Bolivar (2025), as well as to those based on the Cole–Kehoe timing assumption, including Conesa and Kehoe (2017), Bocola and Dovis (2019), Aguiar et al. (2022), and Bianchi and Mondragon (2022), among others. Our analysis contributes to this literature along two main dimensions. First, it shows that the complementarity between the two sources of multiplicity generates plausible combinations of debt service-to-GDP ratios, as well as the level and volatility of interest rate spreads. Second, it provides a coherent account of the events surrounding the European debt crisis in the case of Spain. Several of these papers also explore equilibrium multiplicity in the presence of long-term debt. Long-term bonds naturally reduce the scope for rollover crises and allow for alternative forms of multiplicity, as shown in Aguiar and Amador (2020). For this reason, we focus on the case of one-period debt.[6]

Our paper contributes to our understanding of the forces at play during the European debt crisis. Using models based on fundamentals, Paluszynski (2023) explains the gradual development of that episode, while Paluszynski and Stefanidis (2023) show that frictions in spending adjustment may explain why governments simultaneously increased their external debt. The present paper achieves similar objectives in a model where the debt crisis is self-fulfilling and consistent with the observed effects of the ECB's policy announcements.

## 2 Multiplicity in a three-period model

This section presents a simple three-period environment to illustrate the core mechanism, in which higher interest rates in the first period can expose the borrower to a rollover crisis in the second period, thereby validating the higher rates through a self-fulfilling mechanism.

The borrower receives a deterministic endowment $y$ in each period $t = 0, 1, 2$. It starts with zero debt and can issue one-period non-contingent bonds to competitive, risk-neutral lenders. The borrower cannot commit to repayment. Upon default, it is permanently excluded from international financial markets and consumes $y^d < y$. The risk-free gross interest rate is $R^*$. The borrower chooses debt issuance and default strategies to maximize expected utility with preferences $u(c) = (c^{1-\gamma})/(1-\gamma)$ and discount factor $\beta$. To induce borrowing, we assume $\beta R^* < 1$, so the borrower is more impatient than lenders.

The timing follows Cole and Kehoe (2000). The borrower first issues new debt and then decides whether to repay previously issued debt.[7] This timing introduces rollover risk: if lenders refuse to roll over the debt, the borrower may be forced to default. At the bond auction, we follow Eaton and Gersovitz (1981): the borrower first chooses the amount of resources to raise, denoted by $b$, and lenders then set the gross interest rate $R$, jointly determining debt service $Rb$ in the following period. This assumption generates interest-rate multiplicity as in Calvo (1988). For a given $b$, a higher $R$ increases debt service and thus the incentive to default. In equilibrium, lenders require an expected return equal to

[6] While our main results extend to a version of the model with a moderate profile of long-term debt, a comprehensive analysis of that variant is beyond the scope of this paper.

[7] We do not allow for mixed strategies in the default decision.

$R^*$, so higher default risk feeds back into higher interest rates. Together, these assumptions generate a feedback between beliefs about rollover crises and equilibrium interest rates.

We solve the model by backward induction. In period $t = 2$, the borrower chooses whether to repay inherited debt service $R_2 b_2$ or default. Default occurs if $y^d > y - R_2 b_2$, and repayment otherwise. Define the threshold $\widetilde{B}_2 \equiv (y - y^d)/R^*$. If $b_2 \leq \widetilde{B}_2$, the borrower repays and $R_2 = R^*$; if $b_2 > \widetilde{B}_2$, the borrower defaults with certainty. Thus, $\widetilde{B}_2$ acts as a borrowing limit.

In period $t = 1$, if lenders do not roll over the debt, the borrower defaults if

$$V_1^d \equiv (1+\beta)u(y^d) > u\left(y - R_1 b_1\right) + \beta u\left(y\right).$$

A rollover crisis can occur only if the lack of refinancing triggers default. Define the consumption threshold $\overline{C}$ as the level of consumption in $t = 1$ that makes the borrower indifferent between repayment and default when debt rollover is not possible:[8]

$$(1+\beta)u(y^d) = u\left(\overline{C}\right) + \beta u\left(y\right).$$

Therefore, when debt rollover is not possible, the borrower defaults if debt service is high enough to reduce period-1 consumption below $\overline{C}$. This requires

$$R_1 b_1 > y - \overline{C}. \tag{1}$$

Condition (1) highlights the key mechanism: whether a rollover crisis takes place depends on debt service, $R_1 b_1$, and therefore on the interest rate itself. For a given $b_1$, a higher $R_1$ makes a rollover crisis more likely. In turn, higher rollover risk requires a higher $R_1$ to satisfy lenders' zero-profit condition.

A rollover crisis is equivalent to setting the borrowing limit to zero, a convention we adopt for simplicity. The problem in $t = 1$ becomes

$$V_1\left(R_1 b_1, s_{ck}\right) = \max\{V_1^{nd}\left(R_1 b_1, s_{ck}\right), V_1^d\},$$

[8]For $\gamma \neq 1$, $\overline{C} \equiv \left[(1+\beta)(y^d)^{1-\gamma} - \beta y^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$. For $\gamma = 1$, $\overline{C} = (y^d)^{1+\beta}/y^{\beta}$.

where

$$V_1^{nd}\left(R_1 b_1, s_{ck}\right) = \max_{b_2 \leq \overline{B}_2(s_{ck})} u\left(y - R_1 b_1 + b_2\right) + \beta u\left(y - R^* b_2\right).$$

The sunspot $s_{ck} \in \{0,1\}$ captures market sentiment that drives the rollover risk as in Cole and Kehoe (2000). If $s_{ck} = 0$ and condition (1) holds, a rollover crisis occurs and $\overline{B}_2 = 0$. Otherwise, $\overline{B}_2 = \widetilde{B}_2$.

The borrower's problem in $t = 0$ is

$$V(s_c) = \max_{b_1 \leq \overline{b}} u(y + b_1) + \beta \sum_{s_{ck} \in \{0,1\}} \pi(s_{ck})\, V_1\Big(R_1(b_1, s_c) b_1, s_{ck}\Big),$$

where $\overline{b}$ is an exogenous borrowing limit and $\pi(s_{ck})$ denotes the probability distribution over sunspot realizations. We let $p^{ck}$ denote the probability of the bad sunspot, $p^{ck} \equiv \pi(0)$. The state $s_c \in \{0,1\}$ introduces another sunspot variable that selects among multiple equilibrium interest rates. When multiplicity arises, we follow Ayres et al. (2025) and focus on two extreme equilibria: a pessimistic equilibrium ($s_c = 0$) with the highest feasible $R_1$, and an optimistic equilibrium ($s_c = 1$) with the lowest feasible $R_1$. This implies two equilibrium interest-rate schedules, $R_1(b_1, s_c)$, for $s_c \in \{0,1\}$, reflecting the multiplicity of interest rates consistent with lenders breaking even in expectation.

We now characterize the set of pairs $(b_1, R_1)$ for which lenders break even. This yields two interest-rate schedules and three debt thresholds.

**Proposition 1** *The pairs $(b_1, R_1)$ such that lenders receive an expected return equal to $R^*$, given the borrower's optimal borrowing and default strategies, are:*

*(i)* $b_1 \leq \underline{\underline{B}}_1$ *and* $R_1 = R^*$, *with* $\underline{\underline{B}}_1 \equiv \frac{1}{R^*}\left[y - \overline{C}\right] > 0.$

*(ii)* $\underline{B}_1 \leq b_1 \leq \overline{B}_1$ *and* $R_1 = R^*/(1 - p^{ck})$, *with* $\underline{B}_1 \equiv \frac{1-p^{ck}}{R^*}\left[y - \overline{C}\right] > 0$ *and* $\overline{B}_1$ *defined by*

$$(1+\beta)u(y^d) = u\left(y - \frac{R^*}{1-p^{ck}}\overline{B}_1 + b_2^*\left(\frac{R^*}{1-p^{ck}}\overline{B}_1\right)\right) + \beta u\left(y - R^* b_2^*\left(\frac{R^*}{1-p^{ck}}\overline{B}_1\right)\right),$$

*where*

$$b_2^*\left(x\right) = \min\left\{\frac{(\beta R^*)^{-1/\gamma} y - (y - x)}{1 + (\beta R^*)^{-1/\gamma} R^*}, \widetilde{B}_2\right\}.$$

*Moreover,* $\underline{B}_1 < \underline{\underline{B}}_1$ *for any* $p^{ck} \in (0,1)$ *and* $\underline{B}_1 < \overline{B}_1$.

**Proof** Uncertainty arises solely from the sunspot $s_{ck}$, which equals 0 with probability $p^{ck}$

and 1 with probability $1 - p^{ck}$. There are thus two candidate interest rates: a low rate $R^*$, under which the borrower repays in all states, and a high rate $R^*/(1 - p^{ck})$, under which repayment occurs only if the good sunspot realizes ($s_{ck} = 1$).[9] We characterize the debt thresholds sustaining each equilibrium.

Under $R_1 = R^*$, $\underline{\underline{B}}_1$ is the largest debt level for which condition (1) is not satisfied under the bad sunspot ($s_{ck} = 0$), so the borrower is indifferent between repayment and default when rollover fails. $y > y^d$ implies that $y - \overline{C} > 0$ and therefore $\underline{\underline{B}}_1 > 0$, as a positive debt service is needed to induce default.

Under $R_1 = R^*/(1 - p^{ck})$, $\underline{B}_1$ is the smallest debt level for which (1) holds under the bad sunspot ($s_{ck} = 0$), implying that a rollover crisis induces default in that state. Note that $\underline{B}_1 = (1 - p^{ck})\underline{\underline{B}}_1 < \underline{\underline{B}}_1$ for any $p^{ck} \in (0, 1)$, and that $\underline{B}_1 > 0$.

Next, $\overline{B}_1$ is the largest debt level such that the borrower repays under the good sunspot ($s_{ck} = 1$) at the high interest rate. Let $b_2^*$ denote optimal issuance given inherited debt service $\frac{R^*}{1-p^{ck}}\overline{B}_1$, determined by the minimum of the unconstrained optimum (from the Euler equation) and the borrowing limit $\widetilde{B}_2$. Substituting $b_2^*$ into the repayment value yields the condition defining $\overline{B}_1$.

To show $\underline{B}_1 < \overline{B}_1$, note that

$$u\left(y - \frac{R^*}{1 - p^{ck}}\overline{B}_1 + b_2^*\right) + \beta u\left(y - R^* b_2^*\right) = (1 + \beta)u(y^d) = u\left(y - \frac{R^*}{1 - p^{ck}}\underline{B}_1\right) + \beta u\left(y\right).$$

Optimality of $b_2^*$, with $b_2^* > 0$, implies

$$u\left(y - \frac{R^*}{1 - p^{ck}}\overline{B}_1\right) + \beta u\left(y\right) < u\left(y - \frac{R^*}{1 - p^{ck}}\underline{B}_1\right) + \beta u\left(y\right),$$

and monotonicity of $u$ yields $\overline{B}_1 > \underline{B}_1$. ■

Proposition 1 identifies two regions. Case (i) corresponds to debt levels low enough that repayment occurs in all states. Case (ii) corresponds to intermediate debt levels for which repayment occurs only in good states ($s_{ck} = 1$), giving rise to a higher interest rate. The key insight of the model is that beliefs about rollover risk can be self-fulfilling, as these regions overlap. For intermediate levels of debt, multiple interest rates are consistent with lenders breaking even in expectation. A low interest rate sustains repayment in all states, while a high interest rate induces default in bad states through a rollover crisis. Because higher interest rates increase default risk, and higher default risk in turn justifies higher

[9] Note that $v_1^{nd}(R_1 b_1, 1) \geq v_1^{nd}(R_1 b_1, 0)$.

interest rates, expectations can coordinate on either equilibrium.

Although we assume $\gamma > 0$ in the quantitative analysis, it is instructive to consider the risk-neutral benchmark $\gamma = 0$, for which the thresholds admit closed-form expressions. In this case, $u(c) = c$ and the optimal choice of $b_2$ satisfies $b_2^* = \widetilde{B}_2$, since $\beta R^* < 1$. The thresholds simplify to

$$\underline{\underline{B}}_1 = \frac{(1+\beta)(y-y^d)}{R^*}, \qquad \underline{B}_1 = \frac{(1-p^{ck})(1+\beta)(y-y^d)}{R^*},$$

and

$$\overline{B}_1 = (1-p^{ck})(y-y^d)\left(\frac{1}{R^*} + \frac{1}{(R^*)^2}\right).$$

This benchmark highlights transparently how rollover risk expands the region of multiplicity and how its size depends on fundamentals.

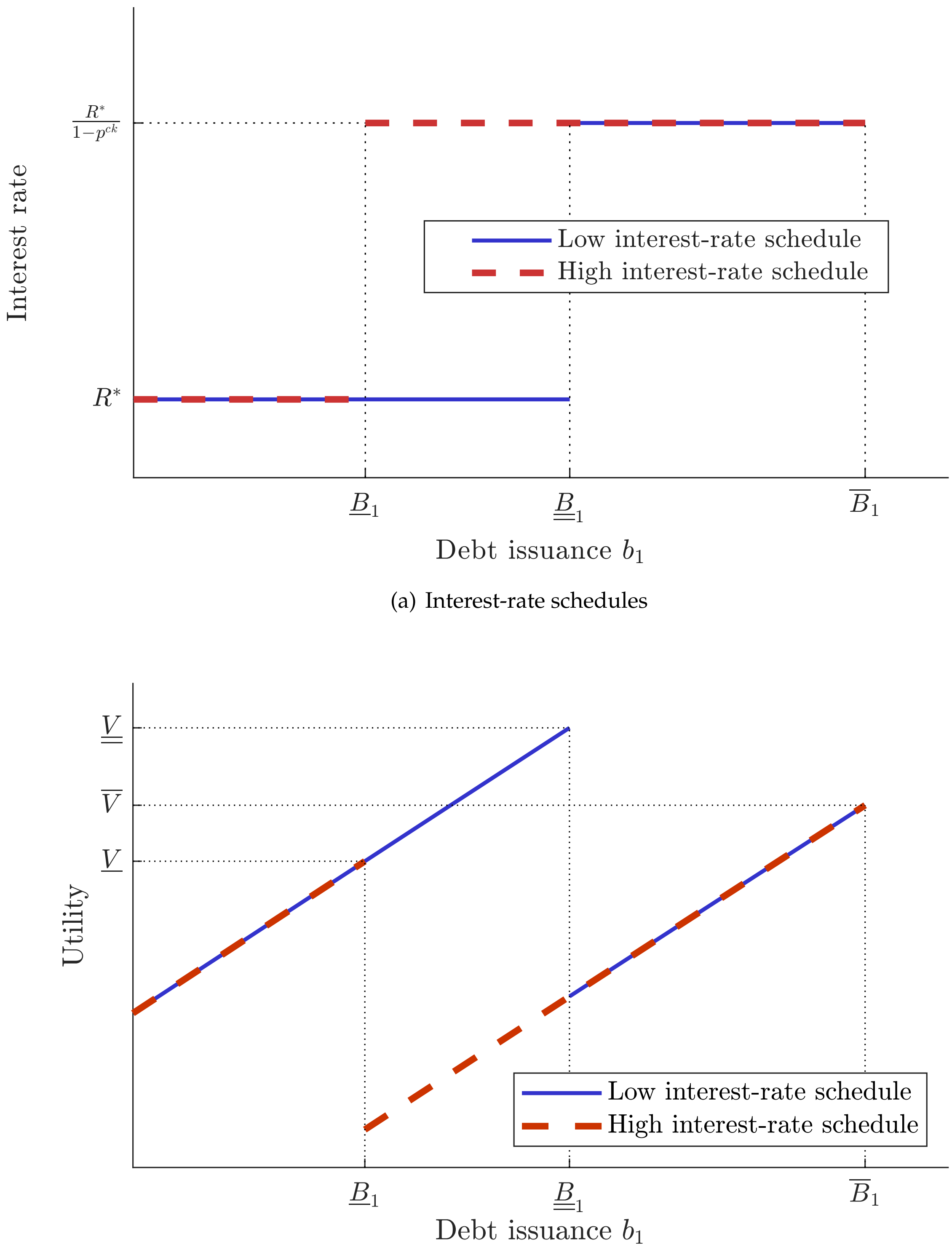
For $\gamma > 0$, closed-form expressions for the thresholds are no longer available. Figure 2(a) illustrates multiplicity using a numerical example.[10] The dashed red line corresponds to the high interest-rate schedule, $R(b_1, s_c = 0)$, while the solid blue line corresponds to the low interest-rate schedule, $R(b_1, s_c = 1)$. Since rollover risk shifts the lower threshold $\underline{B}_1$ below $\underline{\underline{B}}_1$, and $\overline{B}_1 > \underline{B}_1$, there exists a non-empty interval of debt levels, $b_1 \in [\underline{B}_1, \underline{\underline{B}}_1]$, over which both $R_1 = R^*$ and $R_1 = R^*/(1-p^{ck})$ are equilibrium interest rates.

Proposition 1 establishes the multiplicity of interest-rate schedules that arises from the interaction between the rollover risk as in Cole and Kehoe (2000) and interest-rate multiplicity as in Calvo (1988). Next, we consider the borrower's optimal debt issuance in period $t = 0$ under each interest-rate schedule. Figure 2(b) plots the objective function for the numerical example. The selection of interest-rate schedules has first-order implications for both borrowing and welfare. Under the low-interest-rate schedule (solid blue), the borrower chooses $b_1 = \underline{\underline{B}}_1$ and pays $R_1 = R^*$, attaining utility $\underline{\underline{V}}$.

Under the high-interest-rate schedule (dashed red), this allocation is no longer feasible. The borrower faces a trade-off between issuing debt at a higher interest rate, thereby exposing itself to rollover risk in $t = 1$, and limiting borrowing to remain in the safe region with guaranteed repayment. In the numerical example, the former dominates: the borrower optimally chooses $b_1 = \overline{B}_1$ and $R_1 = R^*/(1-p^{ck})$, attaining utility $\overline{V}$.[11]

[10] We set $\beta = 0.4$, $\gamma = 1$, $y = 1$, $y^d = 0.99$, $R^* = 1.0$, and $p^{ck} = 0.15$.

[11] These corner solutions depend on the parameterization and may not arise in alternative calibrations.

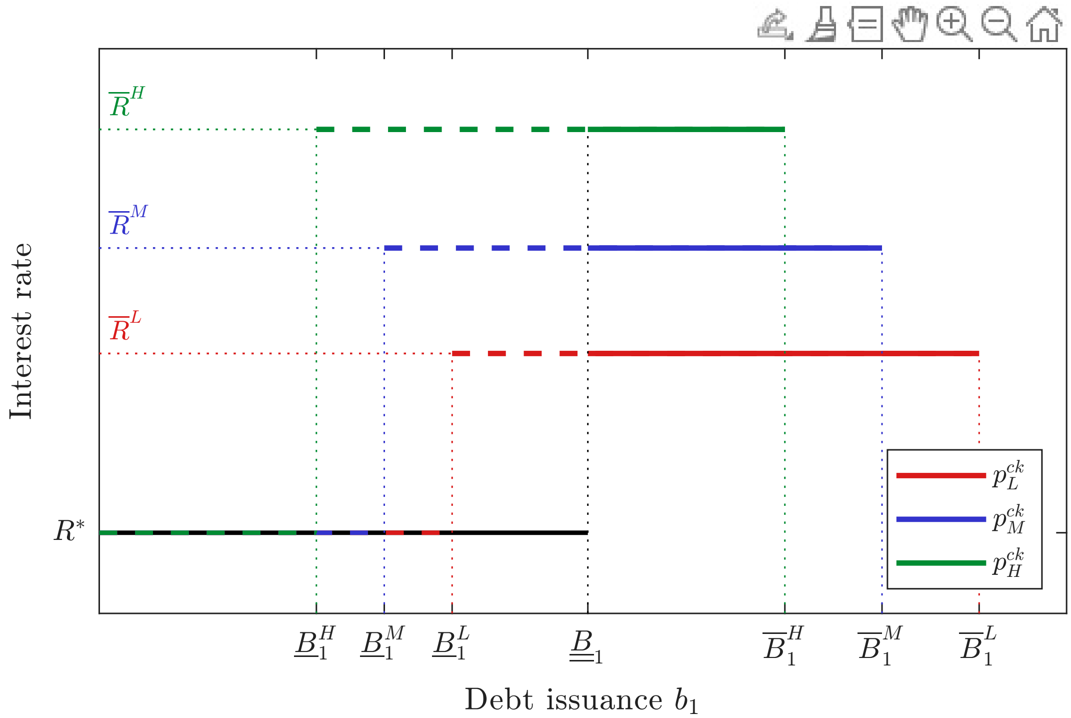



(a) Interest-rate schedules

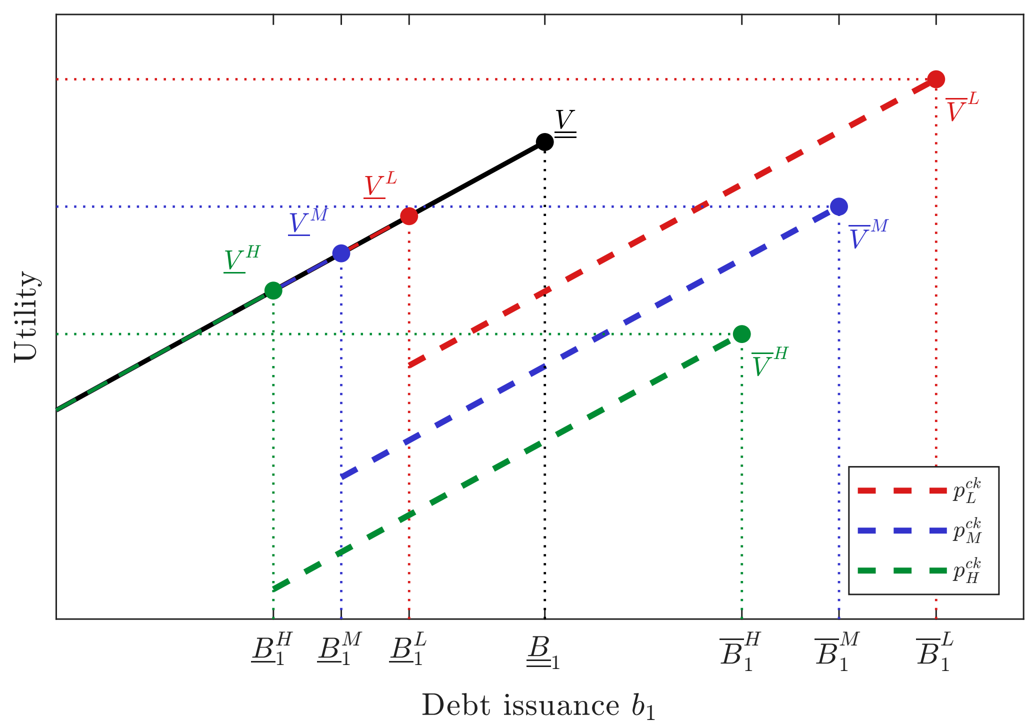


(b) Objective function under high and low interest-rate schedules

Figure 2: Interest-rate schedules and objective functions




(a) Interest-rate schedules




(b) Objective function under high and low interest-rate schedules

Figure 3: Comparative statics with respect to the probability of a rollover crisis

This mechanism has important implications for the quantitative analysis. Fluctuations in the sunspot $s_c$ lead the economy to switch between the high and low interest-rate schedules. Because the high interest-rate schedule is associated with higher debt issuance, these shifts generate endogenous fluctuations in debt and spreads, even in the absence of shocks to fundamentals.

**Varying the probability of rollover crises** Next, we analyze how interest-rate schedules and optimal debt issuance vary with the probability of the sunspot $s_{ck}$, considering low ($p_L^{ck}$), medium ($p_M^{ck}$), and high ($p_H^{ck}$) probabilities.[12] Figure 3(a) shows that the region of multiplicity expands as $p^{ck}$ increases. This is intuitive: a higher probability of the bad sunspot, in which rollover crises may occur, raises the high interest rate, so that $\overline{R}^H > \overline{R}^M > \overline{R}^L$. In turn, higher interest rates increase debt service for a given level of issuance $b_1$, making rollover crises more likely. As shown in Proposition 1, $\underline{\underline{B}}_1$ does not depend on $p^{ck}$, whereas $\underline{B}_1$ decreases with $p^{ck}$, so that $\underline{B}_1^H < \underline{B}_1^M < \underline{B}_1^L$.

Figure 3(b) shows the objective function for different values of this probability. When choosing optimal debt issuance, two opposing forces are at play. On the one hand, higher interest rates increase debt service for any given level of borrowing, making debt less attractive. On the other hand, the maximum level of debt consistent with the low interest rate, $\underline{B}_1$, declines with $p^{ck}$, implying that avoiding the high-rate equilibrium requires lower borrowing and therefore lower current consumption. In our numerical example, as well as in the quantitative analysis that follows, the first effect dominates, and the borrower optimally reduces debt to avoid higher interest rates as the probability of an unfavorable rollover sunspot realization increases.

For $p_L^{ck}$, the borrower chooses to issue high levels of debt at the high interest rate regardless of $s_c$, since $\overline{V}^L > \underline{\underline{V}}$. For $p_M^{ck}$, the borrower issues a high level of debt at the high rate when facing the high-interest-rate schedule ($s_c = 0$), but opts for a lower level of debt at the low rate when facing the low-interest-rate schedule ($s_c = 1$), since $\underline{\underline{V}} > \overline{V}^M > \underline{V}^M$. For $p_H^{ck}$, the high rate is sufficiently elevated that the borrower always prefers to issue less debt at the lower interest rate, regardless of $s_c$; in particular, when $s_c = 0$, this implies even lower debt issuance, given that $\overline{V}^H < \underline{V}^H$.

In Appendix B, we further examine how the multiplicity region and optimal debt issuance

[12] In this example, we use $p_L^{ck} = 0.1$, $p_M^{ck} = 0.15$, and $p_H^{ck} = 0.2$.

vary with changes in the degree of impatience $\beta$ and risk aversion $\gamma$. The role of impatience and risk aversion in shaping optimal debt issuance in the presence of multiple equilibria is well established in quantitative models of self-fulfilling debt crises, such as Aguiar et al. (2022), Ayres et al. (2025), Conesa and Kehoe (2017), and Stangebye (2020), so we relegate the detailed analysis to the appendix.

Intuitively, a more impatient borrower has stronger incentives to borrow more, even if this implies higher debt service in the future, and is therefore more likely to issue debt at higher interest rates. Risk aversion, however, introduces two opposing forces. On the one hand, the borrower has an incentive to issue debt at the risk-free rate in order to avoid uncertainty in consumption in the second subperiod. On the other hand, this is optimal only if it does not require large shifts in consumption across periods, since the risk aversion parameter $\gamma$ also governs the willingness to substitute consumption intertemporally, that is, the preference for smooth consumption paths over time.

Finally, in this simple example we have assumed that income is constant. In a setting in which the borrower expects income to grow in the future, there is an additional incentive to borrow. This force will also be present in our quantitative exercises that incorporate income growth shocks.

# 3 Infinite-horizon model

In this section, we develop an infinite-horizon model to study the interaction between interest-rate multiplicity and rollover risk.

## 3.1 Economic environment

Consider a small open economy with a benevolent sovereign that borrows internationally from competitive lenders and receives a stochastic endowment. Time is discrete and indexed by $t = 0, 1, 2, ....$ Markets are incomplete, and the only asset available for trading is the one-period non-contingent bond. The sovereign lacks commitment to repay its debt. The risk-free gross interest rate is $R^*$. The representative household has preferences given by the expected utility of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{2}$$

where we assume the function $u(\cdot)$ is strictly increasing, concave, and twice continuously differentiable. The discount factor is given by $\beta \in (0,1)$.

**Income process** The economy's income is affected by stochastic endowment growth realizations and evolves according to

$$Y_t = g_t Y_{t-1}, \tag{3}$$

where $g_t$ denotes the growth shock. The growth rate can take two values, $g_L$ and $g_H$, with $g_H > g_L$. It follows a Markov process with the transition probability matrix given by

$$\Pi = \begin{bmatrix} \pi_L & 1-\pi_L \\ 1-\pi_H & \pi_H \end{bmatrix}, \tag{4}$$

where $Pr(g_{t+1} = g_L \mid g_t = g_L) = \pi_L$ and $Pr(g_{t+1} = g_H \mid g_t = g_H) = \pi_H$. We model the income process as a two-regime growth process, following Ayres et al. (2025), who show that a bimodal specification can generate Calvo-style interest-rate multiplicity.[13] That said, Section 3.3 first studies a version of the model without income shocks in order to isolate the mechanism before Section 3.4 introduces stochastic growth.

**Timing** The timing assumptions are the same as in Section 2. The borrower chooses whether or not to default on the debt from the previous period *after* the new debt issuance (Cole and Kehoe, 2000). Similar to Calvo (1988), when the bond auction takes place, the borrower moves first by choosing the amount of resources to raise from bond markets in the current period, denoted by $B'$. Lenders move next and set the gross interest rate $R$ according to an interest-rate schedule, which maps the amount of debt issued and the current state of the economy into a gross interest rate. Shocks are observed at the beginning of the period.

**States** The state variables are $\{A, Y_{-1}, g, \mathbf{s}\}$. $A$ denotes the debt service to be paid in the current period, reflecting the previous-period borrowing decision. It evolves into next period's debt service according to $A' = R(B', Y_{-1}, g, \mathbf{s})B'$, where $R(B', Y_{-1}, g, \mathbf{s})$ denotes the interest-rate schedule. $Y = gY_{-1}$ is current income, and $\mathbf{s} = \{s_c, s_{ck}\}$ is a vector of sunspot realizations corresponding to interest-rate multiplicity and rollover risk, respectively. To make the problem stationary, all value functions are ultimately normalized by

[13] Discreteness is important for generating interest-rate multiplicity, as emphasized by Ayres et al. (2018). In our setting, the model can also accommodate transitory shocks, provided their variance is not too large.

the previous income level $Y_{-1}$.

**Recursive problem** The value function of the government involves a choice of whether or not to default:

$$V(A, Y_{-1}, g, \mathbf{s}) = \max_{d \in \{0,1\}} \left\{ (1-d) V^{nd}(A, Y_{-1}, g, \mathbf{s}) + d V^{d}(Y_{-1}, g, \mathbf{s}) \right\}.$$

The value associated with repayment is

$$V^{nd}(A, Y_{-1}, g, \mathbf{s}) = \max_{B' \leq \overline{B}(A, Y_{-1}, g, \mathbf{s})} \left\{ u(C) + \beta \sum_{g'} \sum_{\mathbf{s}'} \Pi(g'|g) p(\mathbf{s}'|\mathbf{s}) V\Big(B' R(B', Y_{-1}, g, \mathbf{s}), Y, g', \mathbf{s}'\Big) \right\} \quad (5)$$

subject to

$$C = Y - A + B'.$$

The value associated with default is

$$\begin{aligned} V^{d}(Y_{-1}, g, \mathbf{s}) =& u\Big(Y(1-\phi)\Big) \\ &+ \beta \sum_{g'} \sum_{\mathbf{s}'} \Pi(g'|g) p(\mathbf{s}'|\mathbf{s}) \Big\{ \theta V\Big(0, Y, g', \mathbf{s}'\Big) + (1-\theta) V^{d}\Big(Y, g', \mathbf{s}'\Big) \Big\}, \end{aligned}$$

where $\phi$ represents the fraction of income lost upon default. We assume that, following a default, the borrower has probability $\theta$ of being readmitted to capital markets in each period and the recovery rate of defaulted debt is zero.

As in Section 2, the borrowing limit $\overline{B}\left(A, Y_{-1}, g, \mathbf{s}\right)$ equals zero whenever $s_{ck} = 0$, and the lack of new borrowing pushes the country to default. This happens when the following condition is satisfied:

$$u(Y - A) + \beta \sum_{g'} \sum_{\mathbf{s}'} \Pi(g'|g) p(\mathbf{s}'|\mathbf{s}) V\Big(0, Y, g', \mathbf{s}'\Big) < V^{d}(Y_{-1}, g, \mathbf{s}).$$

In equilibrium, the interest-rate schedule $R(B', Y_{-1}, g, \mathbf{s})$ must be such that lenders receive an expected return equal to $R^*$ for each level of borrowing $B'$:

$$R^* = R(B', Y_{-1}, g, \mathbf{s}) \sum_{g'} \sum_{\mathbf{s}'} \Pi(g'|g) p(\mathbf{s}'|\mathbf{s}) \left[1 - d\left(R(B', Y_{-1}, g, \mathbf{s}) B', Y, g', \mathbf{s}'\right)\right].$$

The interest-rate schedule depends on the sunspot variables because, as in Section 2, multiple interest-rate schedules may satisfy the zero-profit condition above, with the sunspot serving as a selection device.[14] Definition 1 formally defines an equilibrium in this economy.

**Definition 1** *A Markov perfect equilibrium for this economy consists of the government value functions* $V(A, Y_{-1}, g, \mathbf{s})$*,* $V^{nd}(A, Y_{-1}, g, \mathbf{s})$*,* $V^{d}(Y_{-1}, g, \mathbf{s})$*; policy functions* $B'(A, Y_{-1}, g, \mathbf{s})$ *and* $d(A, Y_{-1}, g, \mathbf{s})$*; the interest rate schedule* $R(B', Y_{-1}, g, \mathbf{s})$ *and the function for the borrowing limit* $\overline{B}\,(A, Y_{-1}, g, \mathbf{s})$ *such that:*

1. *Policy function* $d(A, Y_{-1}, g, \mathbf{s})$ *solves the government's default-repayment problem.*

2. *Policy functions* $B'(A, Y_{-1}, g, \mathbf{s})$ *solve the government's consumption-saving problem.*

3. *Interest-rate schedules* $R(B', Y_{-1}, g, \mathbf{s})$ *and borrowing limit functions* $\overline{B}\,(A, Y_{-1}, g, \mathbf{s})$ *are such that international lenders receive an expected return equal to* $R^*$*.*

## 3.2 Quantification of the model

In this section, we parameterize the model to evaluate its quantitative performance.[15] The model period is set to one year. We calibrate the version of the model without income shocks to match the experience of Spain over the period 2005–2014. Table 1 summarizes the parameter values. Some parameters are selected externally. Specifically, the gross risk-free rate $R^*$ is set to 1.01.[16] We assume the same CRRA preferences as in Section 2 and set the coefficient of relative risk aversion $\gamma$ to 2, a standard choice in the literature. The probability of re-entry following default, $\theta$, is set to 0.125, implying an average exclusion duration of 8 years, consistent with the evidence in Benjamin and Wright (2013).[17]

The discount factor $\beta$ and the output cost of default $\phi$ are chosen to match key moments from the Spanish data. In particular, we target the average ratio of debt service to GDP in Spain under both the low- and high-spread regimes. For the low-spread regime, we

[14] We follow a selection of interest-rate schedules similar to the one described in Section 2. In the dynamic setup with growth shocks, multiple high-interest-rate segments consistent with the zero-profit condition may emerge. We therefore restrict attention to increasing interest-rate schedules.

[15] We solve a detrended version of the model in Section 3.1, following Aguiar and Gopinath (2006) and Ayres et al. (2025). It is presented in Appendix C.

[16] Between 2005 and 2014, the average annual yield on 10-year U.S. Treasury Inflation-Protected Securities (TIPS) was 1.2%.

[17] Benjamin and Wright (2013) report mean and median duration of default episodes of 8.5 and 7.2 years, respectively.

Table 1: Preset and calibrated parameter values

| **Preset parameters** | | **Value** |
|---|---|---|
| $\gamma$ | Risk aversion | 2 |
| $\theta$ | Re-entry probability | 0.125 |
| $R^*$ | Gross risk-free rate | 1.01 |
| $p^c$ | Probability of bad Calvo sunspot | 0.10 |
| **Calibrated parameters** | | |
| $\beta$ | Discount factor | 0.813 |
| $\phi$ | Income loss in default | 0.022 |
| $p^{ck}$ | Probability of bad Cole–Kehoe sunspot | 0.048 |

target the average of 10.5% observed in 2005–2009, while for the high-spread regime we target the average of 14.5% observed in 2010–2014, as shown in Figure 1. The resulting parameter values are $\beta = 0.813$ and $\phi = 0.022$. Section 3.5 also presents an alternative calibration for Mexico, while Appendix A describes the data sources and the construction of these moments.

The sunspot variables are assumed to follow i.i.d. processes. Let $p^{ck}$ and $p^c$ denote the probabilities that the Cole–Kehoe and Calvo sunspots take the value zero (the pessimistic realization), respectively. As discussed below, $p^{ck}$ governs the high interest rates observed during crises; we calibrate it to match the peak spread of 5.1 percentage points shown in Figure 1, which implies $p^{ck} = 0.048$. For $p^c$, we set the benchmark value to 0.1 and then conduct comparative statics with respect to both probabilities.

## 3.3 No growth shocks

As a first step, we evaluate the model without growth regimes (and thus without fundamental shocks). Income is deterministic and normalized to one in every period. In this setting, rollover risk is the sole driver of defaults and of potential interest-rate multiplicity.

**Multiplicity of interest-rate schedules** Figure 4 illustrates the multiplicity of equilibrium interest rates. The figure shows the pairs of debt issuance and interest rates that satisfy the lenders' zero-profit condition, given the borrower's optimal default strategy. The multiplicity interval confirms our analytical results in Section 2, which show that the Calvo action space can interact with rollover crises to generate overlapping interest-rate schedules even in the absence of income shocks. For levels of debt issuance between $\underline{B}'$ and $\underline{\underline{B}}'$, both the risk-free rate and a high interest rate are consistent with equilibrium. The higher interest rate $\overline{R}$ reflects the probability of default under a rollover crisis in the following period, so that $\overline{R} = R^*/(1 - p^{ck})$. More generally, multiplicity arises for intermediate levels of debt. If debt issuance is sufficiently low, even the high rate does not induce default when rollover fails; for sufficiently high levels of debt, only the high interest rate can be sustained in equilibrium. Given this multiplicity, we follow the strategy described in Section 2 and define a high- (dashed red line) and a low- (solid blue line) interest rate schedule.

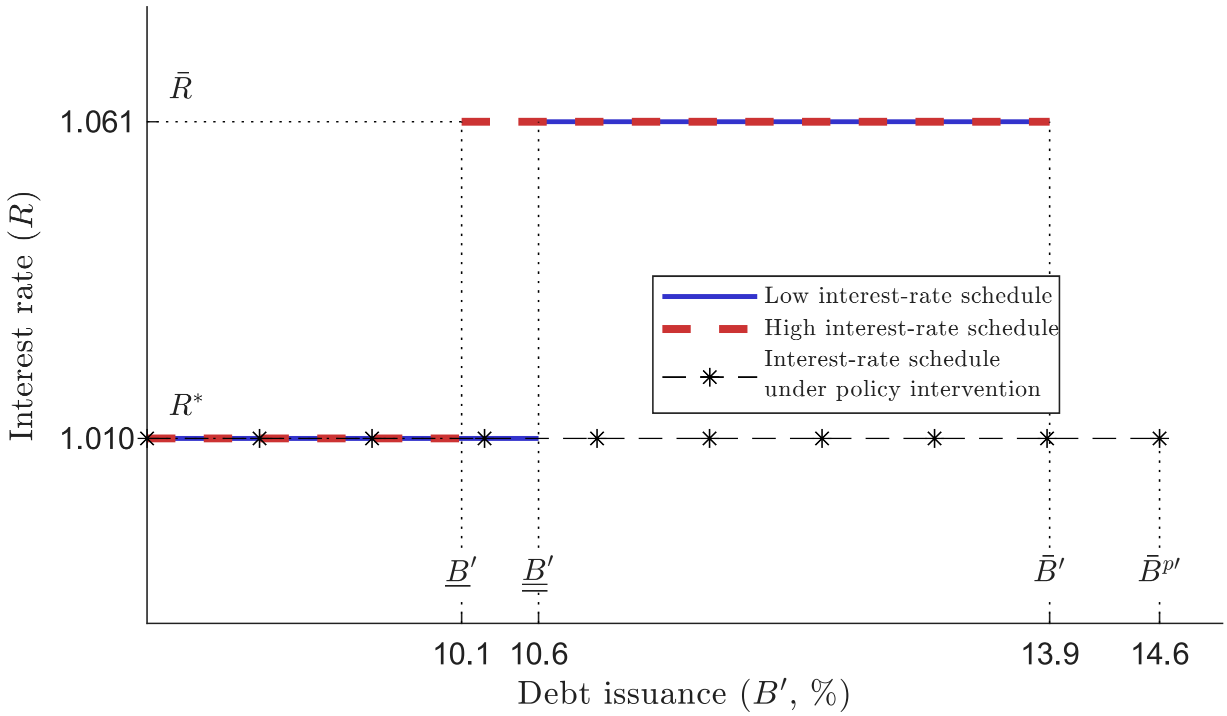



Figure 4: Interest-rate schedules in the deterministic infinite-horizon model

**Optimal debt issuance and model mechanics** Figure 5(a) shows the policy function for debt service, reflecting the optimal choices of debt issuance (Figure 5(b)) and the corresponding interest rates (Figure 5(c)). Together, these figures summarize the model dynamics. Suppose the economy starts with low inherited debt service and faces the high interest-rate schedule (a bad Calvo sunspot). In this case, the economy converges to debt service equal to $\underline{A}$, around 10.2% of GDP. This corresponds to the lowest fixed point of the policy function for debt service under the high interest-rate schedule (dashed red line), where it crosses the 45-degree line in Figure 5(a). This outcome is associated with debt issuance equal to $\underline{B}'$ in Figure 5(b) and the risk-free rate $R^*$ in Figure 5(c), with $\underline{A} = R^* \times \underline{B}'$. At this point, the sovereign refrains from additional borrowing in order to preserve the low interest rate $R^*$.

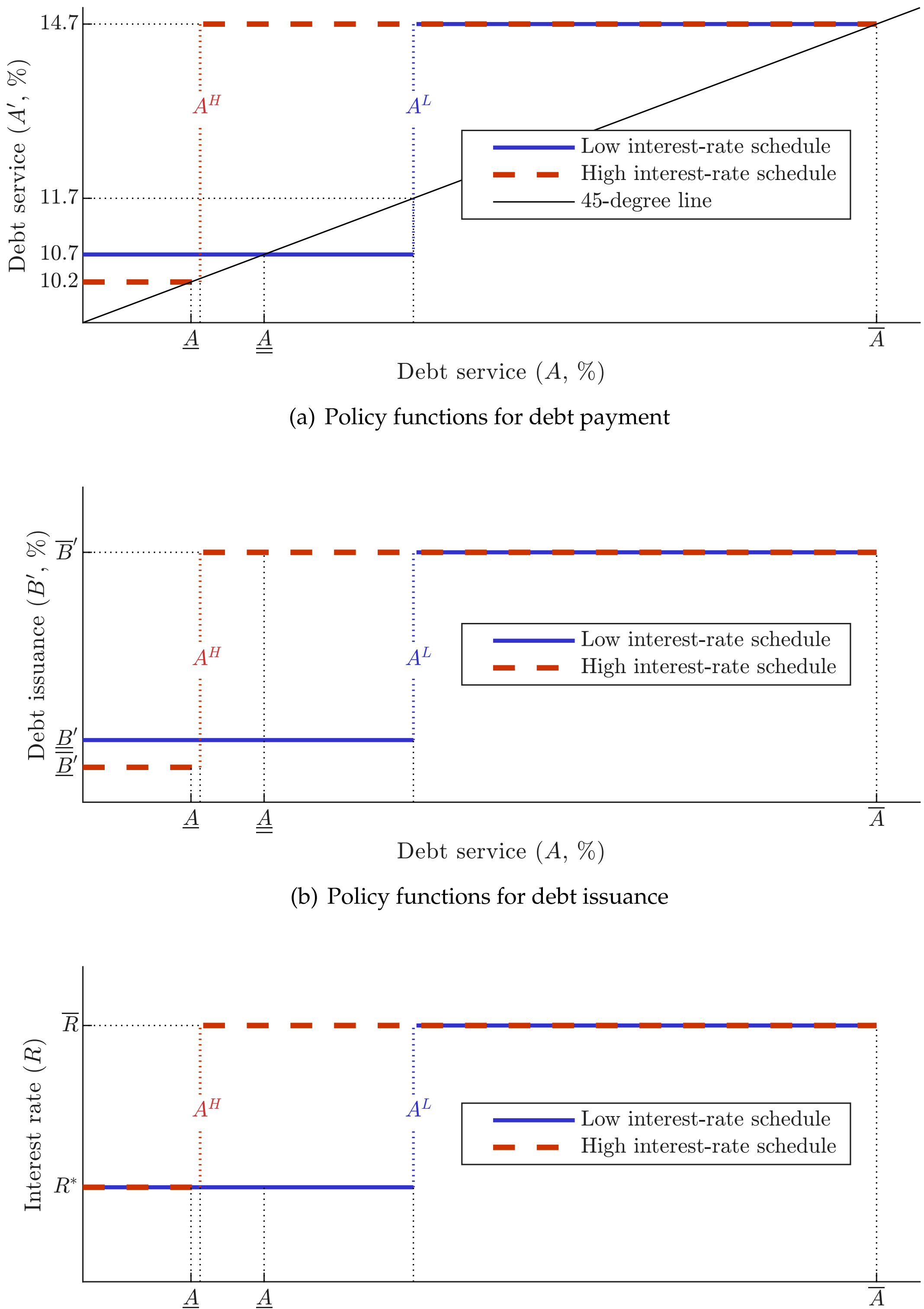
Eventually, a favorable Calvo sunspot is realized, and the sovereign faces the low interest-rate schedule. It then increases borrowing to $\underline{\underline{B}}'$, still at the risk-free rate $R^*$, implying debt service of $\underline{\underline{A}} = R^* \times \underline{\underline{B}}'$, around 10.7% of GDP, which we interpret as capturing Spain's pre-crisis conditions. This corresponds to the lowest fixed point of the policy function for debt service under the low interest-rate schedule (solid blue line), where it crosses the 45-degree line in Figure 5(a).

The key dynamic arises at this point. Once debt service reaches $\underline{\underline{A}}$, a subsequent adverse Calvo sunspot makes it impossible to sustain that level at the low interest rate $R^*$. As in Section 2, the borrower must then choose between reducing borrowing to preserve the lowest interest rate or issuing debt at the higher rate. Under our benchmark calibration, the sovereign opts for the latter, issuing $\overline{B}$ at an interest rate spread $(\overline{R} - R^*)$ of 5.1 percentage points and reaching debt service of $\overline{A} = \overline{R} \times \overline{B}$, around 14.7% of GDP. This captures the buildup in spreads and debt service at the peak of the Spanish debt crisis.[18] Having transitioned to the higher debt-service level $\overline{A}$, the borrower continues to issue debt at the high rate to avoid a sharp contraction in consumption, remaining exposed to rollover crises that eventually materialize.

These dynamics capture the core mechanism of our analysis. Even in the absence of fundamental shocks, the model generates a debt crisis in which the sovereign gradually accumulates debt and faces rising borrowing costs before ultimately defaulting due to a rollover crisis. In this sense, each crisis begins with a gradual increase in debt and spreads following an adverse Calvo sunspot realization, and culminates in default triggered by a bad Cole–Kehoe sunspot—a rollover crisis. Unlike frameworks that rely on fluctuations in fundamentals to produce such dynamics and interest-rate multiplicity, our model shows that they can arise solely from shifts in expectations.

**Policy intervention** We use our framework to analyze policy interventions. First, note that if a lender of last resort can permanently rule out the possibility of a rollover crisis,

[18] This behavior contrasts with that reported in Ayres et al. (2025), also for the case of Spain, where the government responds with endogenous austerity—associated with lower debt issuance and interest rates—thereby preventing the higher interest rates driven by pessimistic expectations from materializing in equilibrium.




(a) Policy functions for debt payment

(b) Policy functions for debt issuance

(c) Policy functions for interest rate

Figure 5: Policy functions

the model becomes degenerate: the impatient government borrows up to the fundamental debt limit and never defaults. As is customary in this class of models, the lender of last resort does not need to intervene in equilibrium. In practice, however, policy institutions face constraints and cannot commit to being continuously available to intervene in debt markets.[19] We therefore move away from this benchmark and consider a policy in which a lender of last resort unexpectedly intervenes and eliminates the possibility of a rollover crisis for one period only.

In this case, the sovereign can borrow at the risk-free rate $R^*$, subject to a borrowing limit that ensures repayment in the following period. This corresponds to the third interest-rate schedule in Figure 4, shown by the black dashed line with asterisks. The borrowing limit is pinned down by the level of debt service $\overline{A}$, above which the sovereign defaults with probability one, regardless of the sunspot realization. Absent the intervention, this level of debt service is associated with the high interest rate $\overline{R}$ and debt issuance $\overline{B}'$. With the intervention, the sovereign instead issues debt at the risk-free rate and increases borrowing to $\overline{B}^{p\prime} = \overline{A}/R^*$, the maximum level of debt issuance along the interest-rate schedule under the policy intervention in Figure 4. Debt service remains at $\overline{A}$, but debt issuance and consumption increase by about 0.7 percentage points of GDP due to the lower interest rate. However, once the policy is withdrawn, the sovereign returns to the high interest rate $\overline{R}$ and reduces issuance to $\overline{B}'$, while maintaining the same level of debt service. Thus, although the intervention provides temporary relief, it induces higher debt issuance during the intervention period and leaves the sovereign in the rollover-crisis region once it is withdrawn.

To induce a more persistent resolution of the debt crisis, an alternative policy is to provide debt relief, reducing debt service to either $A^H$ or $A^L$. The required magnitude of debt relief depends on the realization of the Calvo sunspot. Under a favorable realization (low interest-rate schedule), debt relief must be sufficient to move the economy to debt service equal to $A^L$, approximately 11.7% of GDP, corresponding to a reduction of about 3 percentage points of GDP. From there, the sovereign optimally reduces borrowing further, converging to $\underline{\underline{A}}$ and remaining there unless a bad Calvo sunspot is realized. Under an adverse realization (high interest-rate schedule), a larger reduction is required. In our numerical example, debt relief must be sufficient to bring debt service down to $A^H$ of approximately 10.3% of GDP, close to $\underline{A}$, in order to exit the rollover crisis region, corre-

[19] Ayres et al. (2025) discuss this issue in the context of the European Central Bank during the European debt crisis and the International Monetary Fund in Argentina prior to the 2001 default.

sponding to a reduction of about 4.4 percentage points of GDP. These results highlight that the scale of debt relief needed to restore stability varies with the prevailing interest-rate regime.

The two types of intervention we consider resonate with the aftermath of the Spanish debt crisis in the summer of 2012. As Figure 1 shows, the European Central Bank succeeded in lowering Spanish bond spreads through its "whatever it takes" pledge and the announcement of the OMT program. However, this intervention did not immediately reduce the debt burden. A gradual decline in debt began only in 2015, following the launch of the ECB's asset purchase program, which can be interpreted as a form of implicit debt relief. The precise magnitude of this transfer is difficult to quantify, since the counterfactual path of spreads in the absence of intervention is unobserved. Nonetheless, in Appendix D, we estimate that the present value of this transfer may have ranged from 0.5 to 10 percentage points of Spain's 2025 GDP, based on observed bond purchases and alternative assumptions about the phase-down of the program.[20] Moreover, our model-implied debt relief of 3–4.4 percentage points of GDP, required for the economy to exit the rollover-crisis region, is closely aligned with our median estimate.

The policies discussed so far operate after the country has already issued debt at high interest rates. An alternative approach is to intervene ex ante, preventing the economy from reaching such elevated rates in the first place. In particular, once the economy reaches debt service $\underline{\underline{A}}$, a transfer that reduces debt service to $A^H$ would induce the sovereign to refrain from additional borrowing and preserve the lower interest rate when a bad Calvo-sunspot realization materializes. The fiscal cost of this ex-ante intervention is substantially lower than that of rescuing the government after it enters the rollover crisis region—approximately 0.4 percentage points of GDP, compared to around 3 percentage points in the ex-post case.

**Simulation moments** We also analyze the simulation moments of the benchmark model. As discussed above, the economy transitions primarily across three configurations of debt service, interest rates, and debt issuance: $(\underline{A}, \underline{B}', R^*)$, $(\underline{\underline{A}}, \underline{\underline{B}}', R^*)$, and $(\overline{A}, \overline{B}, \overline{R})$. The probability of the Calvo sunspot, $p^c$, determines how frequently each configuration is visited along the equilibrium path. Column (1) in Table 2 reports statistics from the simulated ergodic distribution of the benchmark economy. The results show that our framework

[20] In a similar and much more comprehensive exercise, Chien et al. (2025) estimate the value of implicit subsidies granted to Spain ranging from 2.1 to 7.2 percent of its GDP.

generates sizable and volatile spreads, with an average of 3.8 percentage points and a standard deviation of 2.2 percentage points. These moments are entirely driven by shifts in market sentiment induced by sunspots. Since output is constant, spreads are acyclical and thus largely detached from fundamentals, except through debt, as higher debt issuance and debt service are associated with higher interest rates.

Table 2: Simulation results: infinite-horizon model without income shocks

| | | Cole–Kehoe sunspot probability | | Preferences | | |
|---|---|---|---|---|---|---|
| | Benchmark (1) | Low $p^{ck}$ (2) | High $p^{ck}$ (3) | High $\gamma$ (4) | Low $\beta$ (5) | High $\beta$ (6) |
| Debt service-to-GDP ratio (%) | 13.6 | 16.5 | 10.6 | 15.0 | 16.7 | 12.5 |
| Average spreads (pp) | 3.8 | 3.7 | 0.0 | 5.1 | 5.1 | 0.0 |
| Standard deviation of spreads (pp) | 2.2 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

*Note: The benchmark assumes probabilities of a bad Cole–Kehoe and Calvo sunspot of $p^{ck} = 4.8\%$ and $p^c = 10\%$, respectively, risk aversion $\gamma = 2$, and discount factor $\beta = 0.813$. The remaining columns report comparative statics: low $p^{ck} = 3.5\%$, high $p^{ck} = 5\%$, high $\gamma = 2.5$, low $\beta = 0.7$, and high $\beta = 0.9$. All statistics exclude periods in which the economy is in default. "pp" denotes percentage points.*

Next, we conduct comparative statics with respect to the probability of a bad realization of the Cole–Kehoe sunspot, $p^{ck}$. The results, reported in columns (2) and (3) of Table 2, are consistent with the mechanisms discussed in Section 2. Column (2) reports the simulation moments when $p^{ck}$ is reduced to 3.5%. In this case, the high interest rate declines, and the sovereign increases borrowing despite facing it. The economy remains in the rollover-crisis region, with the average spread reflecting the probability of a bad Cole–Kehoe realization that triggers a rollover crisis and default. Notably, the standard deviation of spreads is zero, as the economy continuously issues debt at the high interest rate.

In contrast, column (3) reports the results for a higher value of $p^{ck}$, equal to 5%. In this case, the high interest rate becomes sufficiently elevated to deter borrowing into the rollover-crisis region. The economy remains at the low interest rate, implying both zero average spreads and zero volatility.[21]

[21] Section 3.4 shows that the range of Cole–Kehoe sunspot probabilities over which interesting dynamics arise expands considerably when income shocks are introduced.

Figure 6 illustrates how the average and standard deviation of spreads, as well as the average debt-service-to-GDP ratio, vary with the probability of a bad Calvo sunspot, $p^c$. For values of $p^c$ close to zero, the economy remains predominantly at the configuration $(\underline{\underline{A}}, \underline{\underline{B}}', R^*)$, implying an average debt-service-to-GDP ratio of about 10.7%, with zero average and volatility of spreads. As $p^c$ increases, the economy transitions more frequently to the configuration $(\overline{A}, \overline{B}, \overline{R})$, leading to higher spreads and debt service, and thus increasing both the mean and volatility of spreads. For sufficiently high values of $p^c$, the economy spends most of the time in this high-debt, high-spread configuration, with average spreads and debt service converging to $\overline{R}$ and $\overline{A}$, respectively, while the standard deviation of spreads begins to decline toward zero. The peak in spread volatility occurs at $p^c \approx 5\%$, where the standard deviation and average spreads reach about 2.5 and 2.9 percentage points, respectively.

Finally, columns (4), (5), and (6) in Table 2 report the moments of the simulated economy under alternative preference parameters. As the sovereign becomes more risk averse (high $\gamma$, column (4)), we obtain a result consistent with Conesa and Kehoe (2017), in which the sovereign optimally chooses to borrow at the high interest rate and remain exposed to rollover risk in order to smooth consumption across periods. In contrast, columns (5) and

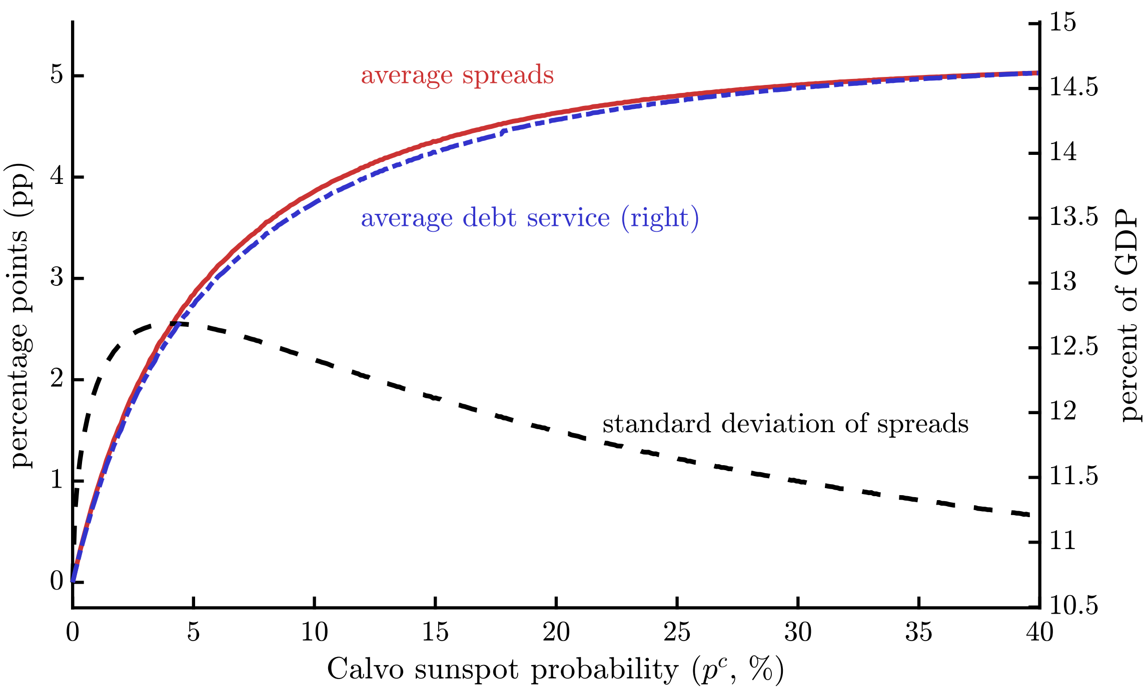



Figure 6: Simulated moments as function of the Calvo sunspot probability

(6) show that when the borrower is more impatient (low $\beta$, column (5)), it tends to borrow more at the high interest rate, while the opposite holds under greater patience (high $\beta$, column (6)). In these cases, the economy converges to either a high- or low-spread equilibrium, with zero volatility in spreads.

Overall, our analysis highlights how the interaction between rollover risk and interest-rate multiplicity, as in Cole and Kehoe (2000) and Calvo (1988), can generate sizable average spreads and volatility even in the absence of fundamental shocks. This represents a departure from the existing literature, which typically relies on income fluctuations—often combined with asymmetric default costs—to generate such dynamics.

The complementarity we emphasize is central to these results. When the probability of a rollover crisis is very low, the model converges to a degenerate outcome in which the borrower issues debt up to the fundamental limit at interest rates close to the risk-free rate, yielding negligible spreads and volatility. Conversely, if the probability of a bad Calvo sunspot is set to zero, the economy becomes trapped in either a high- or low-spread equilibrium, again with no variation in spreads. Thus, the quantitative results hinge on the interaction between these two sources of multiplicity.

One might be tempted to generate additional spread volatility by allowing the probability of a bad Cole–Kehoe realization, $p^{ck}$, to vary over time while shutting down interest-rate multiplicity. However, this would imply that periods of high spreads and elevated rollover risk coincide with lower probabilities of adverse rollover shocks—that is, periods of more favorable market sentiment—as illustrated by the results in columns (1), (2), and (3) of Table 2. In the case of Spain, this would require an improvement in market sentiment (a decline in $p^{ck}$) to generate an increase in spreads and debt service, which is counterfactual.

## 3.4 Growth shocks

In this section, we allow for stochastic growth, as described in the quantitative model in Section 3.1. We use the parameters of the Markov-switching process for Spanish real GDP growth in Ayres et al. (2025). Their estimates for the 1980–2017 period imply high- and low-growth regimes of $g_H = 1.034$ and $g_L = 0.982$, respectively, with persistence parameters $\pi_H = 0.84$ and $\pi_L = 0.63$. Given the estimated income process, and keeping the structural parameter values $\beta$ and $\phi$ from Table 1 unchanged, we then choose the probabilities of bad Calvo and Cole–Kehoe sunspots to match the average debt service and average bond spread over the 1999–2024 period. We find that a Calvo sunspot probability of $p^c = 22.8\%$ and a Cole–Kehoe sunspot probability of $p^{ck} = 1.9\%$ yield average debt-service and spread levels close to the empirical targets. The calibrated parameters are summarized in Table 3.

Table 3: Growth regimes and sunspot probabilities – Spain

| **Preset parameters related to growth regimes** | **Symbol** | **Value** |
|---|---|---|
| High growth rate | $g_H$ | 1.034 |
| Low growth rate | $g_L$ | 0.982 |
| Persistence of high growth | $\pi_H$ | 0.84 |
| Persistence of low growth | $\pi_L$ | 0.63 |
| **Calibrated parameters related to sunspot probabilities** | | |
| Probability of bad Calvo sunspot | $p^c$ | 22.8% |
| Probability of bad Cole–Kehoe sunspot | $p^{ck}$ | 1.9% |

Table 4 presents the simulation results. Column (1) reports the data moments, and Column (2) reports the model-generated moments under the benchmark calibration in Table 3. The results show that varying only the two sunspot probabilities provides enough flexibility to match the targeted moments. The benchmark model matches the average debt service-to-GDP ratio of 10.3% and the average spread of 1 percentage point.

Table 4 also shows that the benchmark model successfully matches the standard deviation of spreads, which was not targeted in this exercise and is a key moment in our analysis. The model delivers a standard deviation of 0.9 percentage points, compared with 1.0 percentage point in the data. The results also show that the model closely matches the countercyclicality of the trade balance. The correlation between the trade balance and GDP growth is $-0.23$ in the benchmark model, compared with $-0.19$ in the data.

The main dimension along which the model is less successful is the cyclicality of spreads. While the correlation between spreads and GDP growth is negative in the data, at $-0.41$, the model delivers a positive correlation of 0.16. This reflects, in part, the persistence of the estimated two-state Markov process for Spain's growth regime together with the

Table 4: Simulation results: infinite-horizon model with income shocks

| | Data (1) | Benchmark model (2) | No sunspot (3) | Only Calvo sunspot (4) | Only Cole–Kehoe sunspot (5) |
|---|---|---|---|---|---|
| **Targeted moments** | | | | | |
| Debt service-to-GDP ratio (%) | 10.3 | 10.3 | 23.9 | 15.8 | 21.1 |
| Average spreads (pp) | 1.0 | 1.0 | 0.0 | 1.0 | 1.0 |
| **Untargeted moments** | | | | | |
| Standard deviation of spreads (pp) | 1.0 | 0.9 | 0.0 | 0.8 | 0.0 |
| Correlation of | | | | | |
| trade balance and GDP growth | -0.19 | -0.23 | -1.0 | -0.41 | -1.0 |
| spreads and GDP growth | -0.41 | 0.16 | 0.0 | 0.85 | 0.0 |
| spreads and trade balance | 0.57 | -0.12 | 0.0 | -0.46 | 0.0 |

*Note: All statistics exclude periods in which the economy is in default. "pp" denotes percentage points. Column (2) corresponds to the benchmark model under the calibration in Tables 1 and 3, with $p^c$ and $p^{ck}$ from Table 3. In the remaining columns, only the sunspot probabilities are changed. In Column (3), $p^c = 0\%$ and $p^{ck} = 0\%$. In Column (4), $p^c = 9.5\%$ and $p^{ck} = 0\%$. In Column (5), $p^c = 0\%$ and $p^{ck} = 1.0\%$. For Columns (4) and (5), the sunspot probabilities are chosen so that the model matches the average spread in the data. Trade balance denotes the trade balance-to-GDP ratio. Appendix A describes the data sources and the computation of the data moments.*

low average spread target. In the low-growth regime, the interest-rate schedule features higher interest rates, since there is a high probability that the economy will remain in the low-growth state in the following period, when default is more likely. In equilibrium, the borrower therefore reduces issuance in low-growth regimes in order to avoid high interest rates, accumulating debt primarily in high-growth regimes. As a result, most defaults associated with adverse income realizations occur when the economy transitions from high to low growth. This mechanism is amplified by the assumption that the borrower defaults on all debt obligations, so spreads fully reflect the probability of entering a low-growth regime in the future.[22] Section 3.5 presents quantitative results for the model calibrated to Mexico and shows that the lower persistence of the low-growth regime in that case reduces the correlation between spreads and GDP growth.

This limitation should also be interpreted in light of the main mechanism of the model. Income shocks affect the government's borrowing decisions, as reflected in the negative

[22]This issue could be mitigated by assuming some debt recovery by creditors, which would lower overall spreads and could induce the borrower to choose higher spreads also in low-growth regimes.

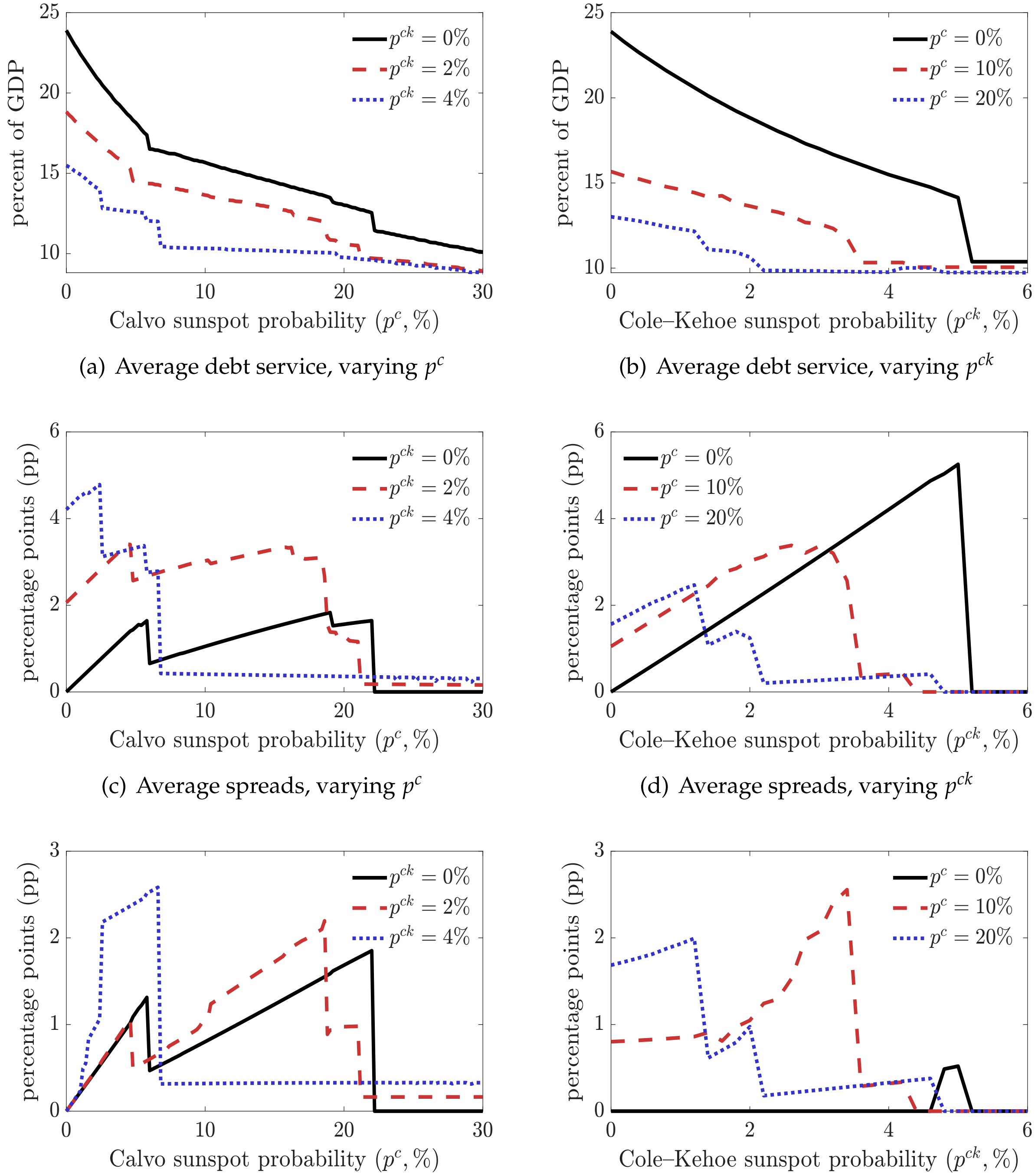
correlation between income and the trade balance, but they are not the primary driver of debt crises in the model; sunspots are. We therefore maintain the simpler framework in order to highlight the interaction between =rollover and interest-rate risks.

To highlight the role of sunspots in the quantitative results, Columns (3), (4), and (5) in Table 4 report the moments generated by versions of the benchmark model in which we vary the probabilities of the sunspot variables. Column (3) reports the case in which both sunspot probabilities are set to zero, $p^c = p^{ck} = 0$, effectively eliminating any role for shifts in creditors' beliefs in equilibrium outcomes. In this case, both the average spread and the standard deviation of spreads fall to zero, while average debt service increases to 23.9% of GDP. This illustrates that, in the benchmark calibration, multiplicity plays a key role in allowing the model to match the targeted data moments.

Columns (4) and (5) in Table 4 then show that the benchmark model relies on both sunspots operating together, highlighting their complementarity. Column (4) reports the case in which rollover risk is shut down. That is, the probability of the Cole–Kehoe sunspot, $p^{ck}$, is set to zero, and the probability of the Calvo sunspot, $p^c$, is chosen to match the average spread in the data. In this case, while the model is still able to generate spread volatility close to that observed in the data, it can no longer match the debt service-to-GDP ratio, delivering an average that is significantly higher than in the data.

Column (5) reports a similar exercise for the case in which the probability of the Calvo sunspot is set to zero, while the probability of the Cole–Kehoe sunspot is chosen to match average spreads in the data. Again, the model is capable of matching the targeted moment, but it generates an average debt service-to-GDP ratio that is twice as high as in the data, while generating no volatility in interest rate spreads. Together, these results demonstrate the role of the complementarity between the two sources of multiplicity in accounting for the quantitative results of the benchmark model.

**Comparative statics** Figure 7 further illustrates how average debt service-to-GDP, average spreads, and the standard deviation of spreads vary with the Calvo and Cole–Kehoe sunspot probabilities. The three rows report average debt service, average spreads, and spread volatility, respectively. The left column varies the Calvo sunspot probability, $p^c$, from 0% to 30%, for three values of the Cole–Kehoe sunspot probability: $p^{ck} = 0\%$ (solid black line), $p^{ck} = 2\%$ (dashed red line), and $p^{ck} = 4\%$ (dotted blue line). The right column varies the Cole–Kehoe sunspot probability, $p^{ck}$, from 0% to 6%, for three values of




(a) Average debt service, varying $p^c$

(b) Average debt service, varying $p^{ck}$

(c) Average spreads, varying $p^c$

(d) Average spreads, varying $p^{ck}$

(e) Standard deviation of spreads, varying $p^c$

(f) Standard deviation of spreads, varying $p^{ck}$

Figure 7: Comparative statics over sunspot probabilities

the Calvo sunspot probability: $p^c = 0\%$ (solid black line), $p^c = 10\%$ (dashed red line), and $p^c = 20\%$ (dotted blue line). Therefore, the black lines correspond to either the pure-Calvo (left Column) or the pure Cole–Kehoe (right column) variants of the model.

Figures 7(a) and 7(b) show that increasing the sunspot probabilities reduces debt service in equilibrium, as borrowing becomes more restricted. Similar to the results in Section 3.3, average spreads tend to increase at low sunspot probabilities, as shown in Figures 7(c) and 7(d). Eventually, however, the probability of a crisis becomes high enough that the borrower refrains from borrowing at higher interest rates, and average spreads fall to zero. A similar nonmonotonic pattern appears for the standard deviation of spreads in Figures 7(e) and 7(f).

Overall, Figure 7 illustrates how different configurations of the sunspot probabilities $p^c$ and $p^{ck}$ can generate a wide range of moments through the complementarity between the two sources of multiplicity. To see this complementarity, notice first that for the Calvo sunspot probabilities up to around 0.05, the average spread increases roughly by the probability of a rollover crisis, while the standard deviation of the spread is the same (as the pure Cole-Kehoe model does not generate any volatility in the bond spread). This implies that the two sources of multiplicity in this region are independent of each other and their effects are approximately "additive". Beyond this threshold, however, an interesting interaction occurs. As we add rollover crises, the average spread rises by less than the probability of a bad Cole-Kehoe sunspot, while the standard deviation of the spread rises by more. At the peak, the standard deviation reaches 2.5% in our model, compared to 1.8% in the pure Calvo and zero volatility in the pure Cole-Kehoe variants. Hence, the interaction between the two sunspots in this region can be thought of as "multiplicative".

### 3.5 The Case of Mexico

In this last section, we extend our analysis to the case of an emerging economy, Mexico, which allows for a comparison of our mechanism and quantitative results to other benchmark studies in the literature. Mexico features higher debt service-to-GDP ratios, higher volatility in GDP growth, and higher and more volatile spreads. The exercise shows that the complementarity between rollover and interest-rate risks is relevant not only for interpreting the European crisis, but also for matching standard emerging-market moments. Following Aguiar et al. (2022), we calibrate the model at a quarterly frequency.

First, we estimate the income process for Mexico using GDP data from 1994Q1 to 2024Q4, matching the availability of Mexico's bond spread data, as described in Appendix A. We apply the filter of Kim (1994) and estimate the parameters of the two-state Markov process for Mexico's growth regimes by maximum likelihood. The resulting estimates are $g_H = 1.015$, $g_L = 0.987$, $\pi_H = 0.67$, and $\pi_L = 0.39$. Second, following Aguiar et al. (2022), we set the discount factor $\beta$ to 0.8, the risk-free rate $R^*$ to 1.01, the risk aversion parameter $\gamma$ to 2, and the re-entry probability $\theta$ to 0.125. Third, we calibrate the default cost $\phi$, together with the two sunspot probabilities, to match three moments of interest: the average and standard deviation of interest rate spreads and the average debt service-to-GDP ratio.[23] This represents a departure from the previous section, where the standard deviation of spreads was not a targeted moment. The inferred parameter values are $\phi = 0.039$, $p^c = 26.6\%$, and $p^{ck} = 1.1\%$. Hence, relative to the calibration for Spain, Mexico requires a higher default cost and higher probabilities of bad realizations of both the Calvo and Cole–Kehoe sunspots. These probabilities are expressed at a quarterly frequency. Table 5 summarizes the parameter values for the case of Mexico.

Table 5: Calibration – Mexico (quarterly frequency)

| **Preset parameters** | **Symbol** | **Value** |
|---|---|---|
| Discount factor | $\beta$ | 0.8 |
| Income loss in default | $\phi$ | 0.039 |
| High growth rate | $g_H$ | 1.015 |
| Low growth rate | $g_L$ | 0.987 |
| Persistence of high growth | $\pi_H$ | 0.67 |
| Persistence of low growth | $\pi_L$ | 0.39 |
| **Calibrated parameters related to sunspot probabilities** | | |
| Probability of bad Calvo sunspot | $p^c$ | 26.6% |
| Probability of bad Cole–Kehoe sunspot | $p^{ck}$ | 1.1% |

The simulation results are reported in Table 6. Column (1) reports the data moments, and Column (2) reports the model-implied moments under the benchmark calibration. As in the case of Spain, the results show that the model has enough flexibility to match the three

[23] Appendix A describes the details of the data construction. One notable departure from the calibration in Aguiar et al. (2022) is that we target average debt service-to-GDP over a long time series, whereas their model targets debt service due at the height of the 1994 crisis as an outlier realization. We make this departure because we do not have reliable data for that episode.

targeted moments. The model matches the average debt service-to-GDP ratio of 18.9%, as well as the average spread of 3.4 percentage points and the standard deviation of spreads of 2.0 percentage points, both computed after annualizing the quarterly spreads. The results in Column (2) also show that spreads are acyclical under the benchmark calibration, while the correlations of the trade balance with spreads and GDP are both negative.

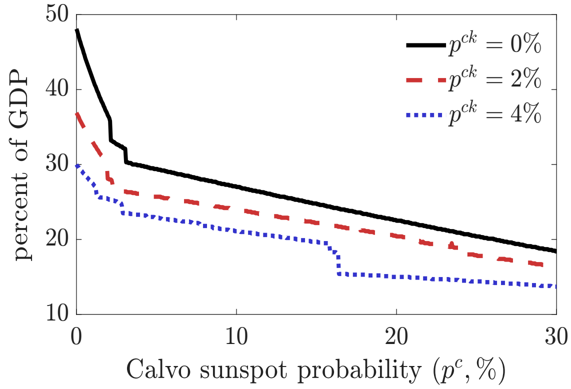
Table 6: Simulation results for the case of Mexico (quarterly frequency)

| | Data (1) | Benchmark model (2) | No sunspot (3) | Only Calvo sunspot (4) | Only Cole–Kehoe sunspot (5) |
|---|---|---|---|---|---|
| Debt service-to-GDP ratio (%) | 18.9 | 18.9 | 48.1 | 33.0 | 43.0 |
| Average spreads (pp, annualized) | 3.4 | 3.4 | 0.0 | 3.4 | 3.4 |
| Standard deviation of spreads (pp, annualized) | 2.0 | 2.0 | 0.0 | 0.1 | 0.0 |
| Correlation of | | | | | |
| trade balance and GDP | 0.34 | -0.06 | -1.0 | -0.22 | -1.0 |
| spreads and GDP | -0.13 | 0.0 | 0.0 | -0.46 | 0.0 |
| spreads and trade balance | 0.38 | -0.71 | 0.0 | 0.08 | 0.0 |

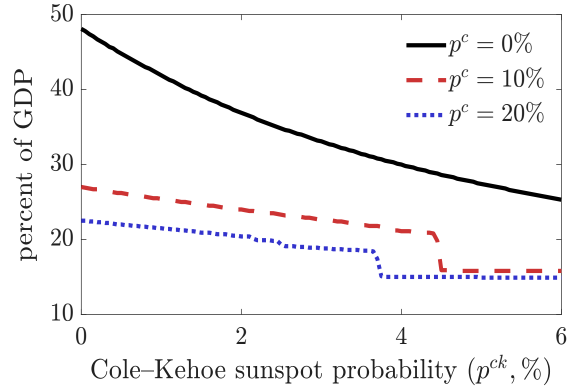
*Note: All statistics exclude periods in which the economy is in default. "pp" denotes percentage points. Column (2) corresponds to the benchmark model under the calibration in Table 5. In the remaining columns, only the sunspot probabilities are changed. In Column (3), $p^c = 0\%$ and $p^{ck} = 0\%$. In Column (4), $p^c = 2.3\%$ and $p^{ck} = 0\%$. In Column (5), $p^c = 0\%$ and $p^{ck} = 0.8\%$. For Columns (4) and (5), the sunspot probabilities are chosen so that the model matches the average spread in the data. Trade balance denotes the trade balance-to-GDP ratio. Appendix A describes the data sources and the computation of the data moments.*

Following the same strategy as in Section 3.4, Columns (3)–(5) present the results under different values of the sunspot probabilities. Column (3) reports the results for the quantitative model when both sunspot probabilities are set to zero, $p^c = p^{ck} = 0$. In this case, both the average and standard deviation of spreads collapse to zero, while the average debt service-to-GDP ratio rises to 48.1%. Again, this highlights how the sources of multiplicity we explore greatly affect these moments.

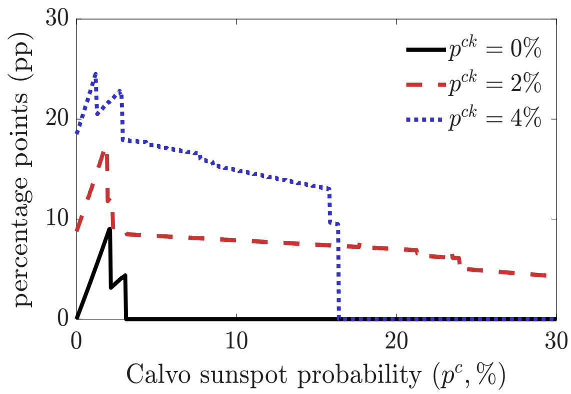
To show that the interaction between rollover risk and interest-rate risk is key for the results, Columns (4) and (5) report the simulation results for the cases in which we recalibrate one sunspot probability to match the average spread in the data while setting the other sunspot probability to zero. In these cases, while the model is able to match the average spread, the standard deviation of spreads also collapses to close to zero, while



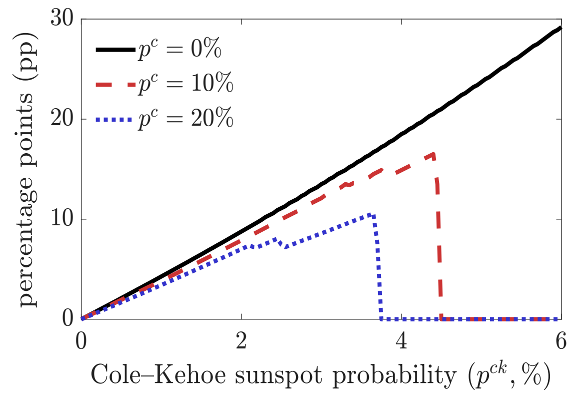

(a) Average debt service, varying $p^c$



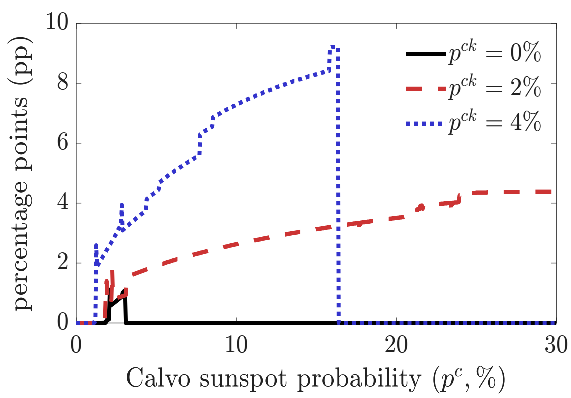

(b) Average debt service, varying $p^{ck}$



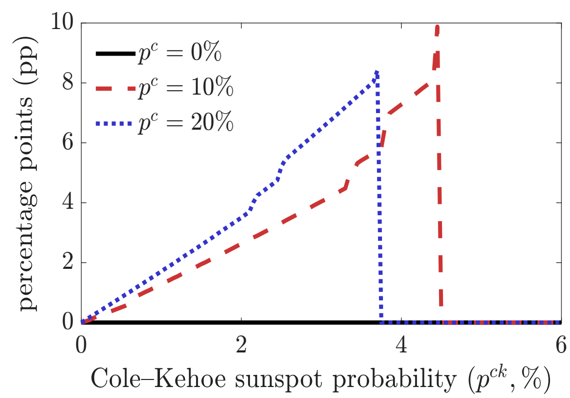

(c) Average spreads, varying $p^c$




(d) Average spreads, varying $p^{ck}$




(e) Standard deviation of spreads, varying $p^c$




(f) Standard deviation of spreads, varying $p^{ck}$

Figure 8: The case of Mexico: comparative statics over sunspot probabilities

the average debt service-to-GDP ratio increases significantly.

Figure 8 reports comparative statics with respect to the Calvo and Cole–Kehoe sunspot probabilities for the Mexico calibration. The layout mirrors Figure 7: the rows report average debt service-to-GDP, average spreads, and the standard deviation of spreads, while the columns vary $p^c$ and $p^{ck}$, respectively. The figure shows that the main message from Spain carries over to Mexico. Different configurations of the two sunspot probabilities generate a wide range of outcomes for debt service, average spreads, and spread volatility. Moreover, as in the case of Spain, the joint presence of rollover and interest-rate risks generates outcomes that neither source of multiplicity can account for on its own. In particular, the complementarity can be "additive" in nature (for Calvo sunspot probabilities up to about 3%) or "multiplicative" (beyond that threshold). Overall, the Mexico calibration reinforces the quantitative role of the complementarity between rollover and interest-rate risks.

# 4 Conclusion

This paper contributes to the literature on self-fulfilling debt crises by introducing a simple model in which belief-driven runs on government debt generate interest-rate multiplicity. In the model, pessimistic beliefs can lead to high interest rates, which in turn validate the initial pessimism by increasing debt service and making default more likely. The main feature of the framework is that it generates rich dynamics for sovereign debt and interest-rate spreads through the complementarity between the mechanisms underlying slow- and fast-moving debt crises, *even in the absence of shocks to fundamentals*. Through its combination of simplicity and quantitative discipline, the model provides a unified way to interpret the slow- and fast-moving stages of the European debt crisis of 2008–2012, as illustrated by our case study of Spain.

When augmented with income growth shocks, the benchmark quantitative model matches key moments in the data for Spain, including average debt service, average spreads, and spread volatility. The calibration to Mexico further shows that the mechanism is not specific to the European debt crisis, but can also account for standard emerging-market debt and spread moments. Across both applications, the results show that this simple framework can generate a wide range of outcomes under different configurations of the sunspot probabilities, highlighting the quantitative importance of the interaction between rollover and interest-rate risks.

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# Appendices for Online Publication

# A Data Sources and Target Moments

This section describes the data sources and the calculation of the statistics reported in the main text.

## A.1 Bond Spreads

To calculate bond spreads for Spain, we use interest rates on long-term Spanish government bonds from the OECD, covering the period from 1999, when the euro was introduced, through 2024. We then subtract the yield on long-term German government bonds. Over this period, both the mean and the standard deviation of spreads are 1 percentage point.

To obtain bond spread data for Mexico, we use the Emerging Market Bond Index series, available at quarterly frequency from 1994 through 2024. Over this period, the mean bond spread is 3.4 percentage points, while the standard deviation is 2.0 percentage points.

## A.2 Debt

**Spain** Data on debt service for Spain are not readily available. Following the same procedure as Ayres et al. (2025), we impute debt service as the sum of interest payments due during the year, short-term debt maturing during the year, and the fraction of long-term debt that matures during the year.

1. Interest payments: We use data on net investment income from the balance of payments. This is computed as investment payments (series code BE 17 5A.7) minus investment income (series code BE 17 5A.1), both from the Banco de España.

2. Short-term debt: We use data on net short-term debt related to portfolio investment, excluding the Banco de España. This is computed as liabilities (series code BE 17 27.8) minus assets (series code BE 17 22.8), both from the Banco de España.

3. Long-term debt maturing: We compute the stock of long-term debt as total debt minus short-term debt, both as described above. We multiply this amount by $\delta = 0.15$ to estimate principal payments on long-term debt. Since we do not have data on the average maturity of Spain's net external debt, we use $\delta = 0.15$, which corresponds

to an average maturity of 6 to 7 years, in line with Spain's central government debt maturity as reported by the Tesoro Público de España.

**Mexico** Data on government debt for Mexico come from the World Bank's International Debt Statistics. This source separately reports debt service and short-term debt in U.S. dollars at annual frequency. Because our calibration for Mexico is quarterly, we proceed as follows. Short-term debt is a stock variable, so we treat it as such and divide it by Mexico's quarterly GDP, effectively multiplying its annual GDP ratio by four. Debt service, by contrast, is a flow variable, so we assume that annual payments are evenly spread across the four quarters. We then divide this amount by quarterly GDP and add the resulting ratio to the short-term debt ratio. Averaging the series over the period 1994Q1–2024Q4, the same window as the EMBI bond spread data, yields a debt-service-to-GDP ratio of 18.9%.

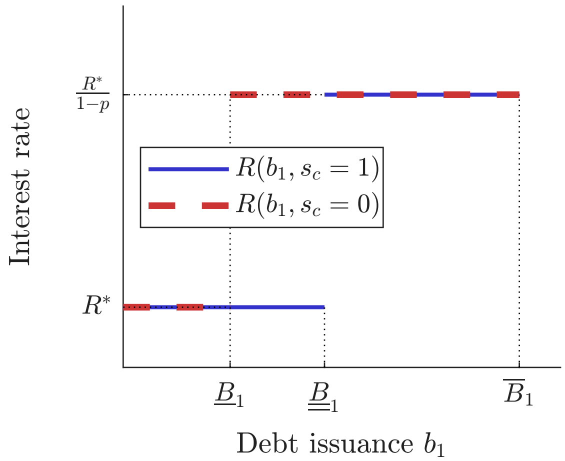
# B Additional Comparative Statics for the Three-Period Model

This section provides additional comparative statics for the three-period model introduced in Section 2. Figure 9 displays the equilibrium interest-rate schedules and objective functions for $\beta = 0.3, 0.4, 0.5$, with both axes held fixed across panels to ensure comparability. As the borrower becomes more impatient (lower $\beta$), the set of debt levels consistent with the low interest rate shrinks, while the range of debt levels for which the high interest rate can be sustained expands. Consequently, the multiplicity region shifts toward lower levels of debt issuance.

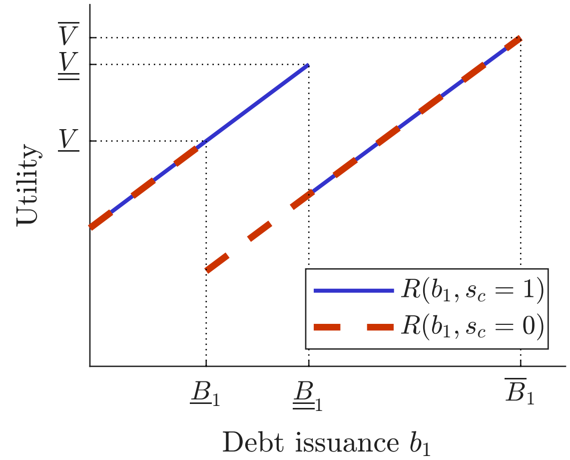
Turning to the objective functions, lower values of $\beta$ increase the borrower's willingness to shift consumption toward the present, raising the attractiveness of borrowing despite higher future repayment obligations. For $\beta = 0.3$, the borrower optimally chooses the highest feasible debt level, $\overline{B}_1$, associated with the high interest rate, regardless of which interest rate schedule is realized. As $\beta$ increases, borrowing behavior becomes more sensitive to the prevailing interest rate. For $\beta = 0.4$, the borrower selects high debt levels when facing the high-interest-rate schedule, but opts for more moderate borrowing under the low-interest-rate schedule. For $\beta = 0.5$, the borrower prefers to limit borrowing in order to remain in the low-interest-rate region. In this case, equilibrium selection affects borrowing only through the level of debt, with the pessimistic equilibrium (high interest rate) associated with lower issuance.

We next examine how outcomes vary with the degree of risk aversion in Figure 10. Higher values of $\gamma$ correspond to stronger aversion to fluctuations in consumption. This introduces a trade-off: issuing debt at the risk-free rate eliminates uncertainty in the second subperiod but may require greater intertemporal variation in consumption. Figures 10(a) and 10(c) compare the equilibrium interest-rate schedules for the benchmark case, $\gamma = 1$, and for a high level of risk aversion, $\gamma = 15$. The overall shape of the schedules is largely unchanged. While greater risk aversion slightly reduces the maximum debt level consistent with the low interest rate, the magnitude of this effect is small relative to the change in $\gamma$.

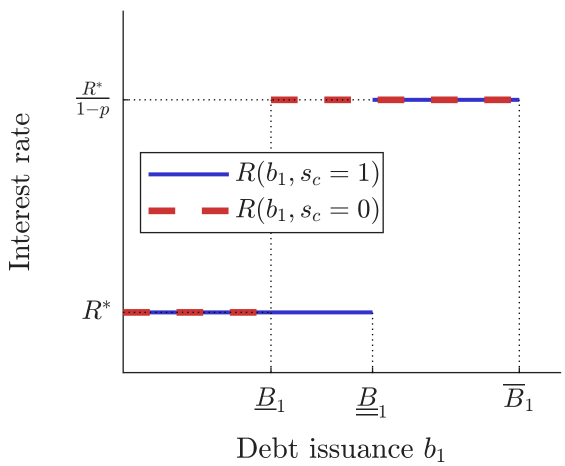
By contrast, risk aversion has a more pronounced effect on the objective function and, therefore, on optimal borrowing choices. As $\gamma$ increases, the borrower places greater weight on consumption smoothing, which in our numerical example leads to more conservative borrowing behavior and a preference for avoiding the high-interest-rate region.



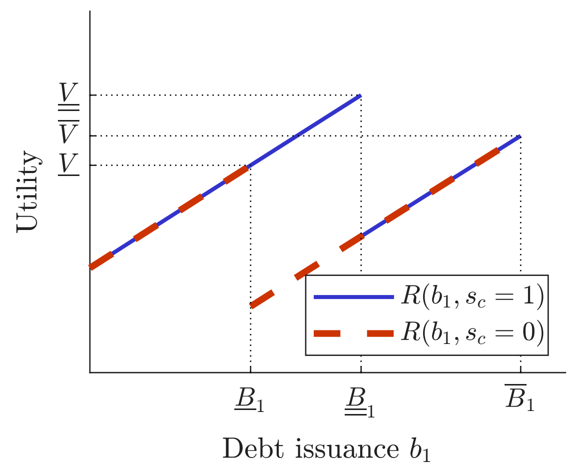

(a) Interest-rate schedule: $\beta = 0.3$



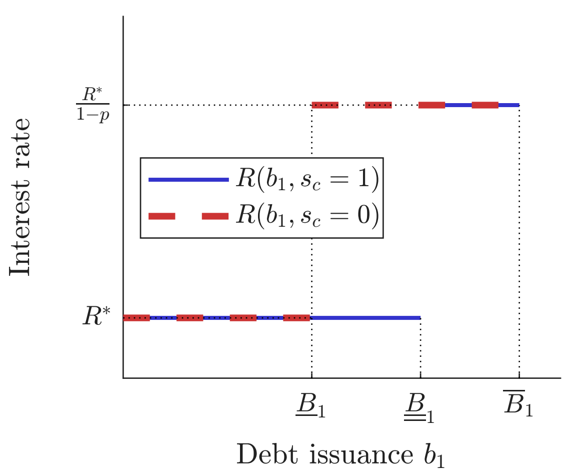

(b) Objective function: $\beta = 0.3$



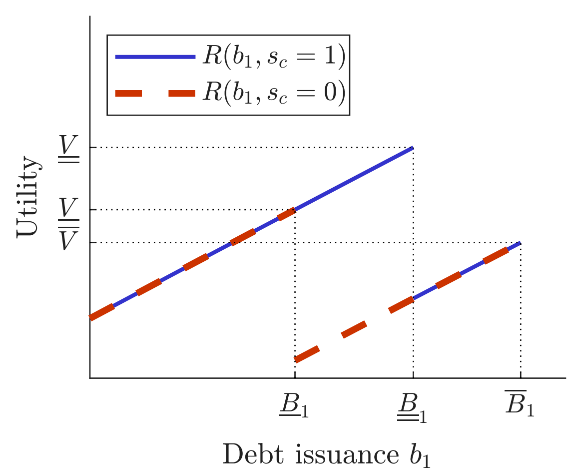

(c) Interest-rate schedule: $\beta = 0.4$




(d) Objective function: $\beta = 0.4$



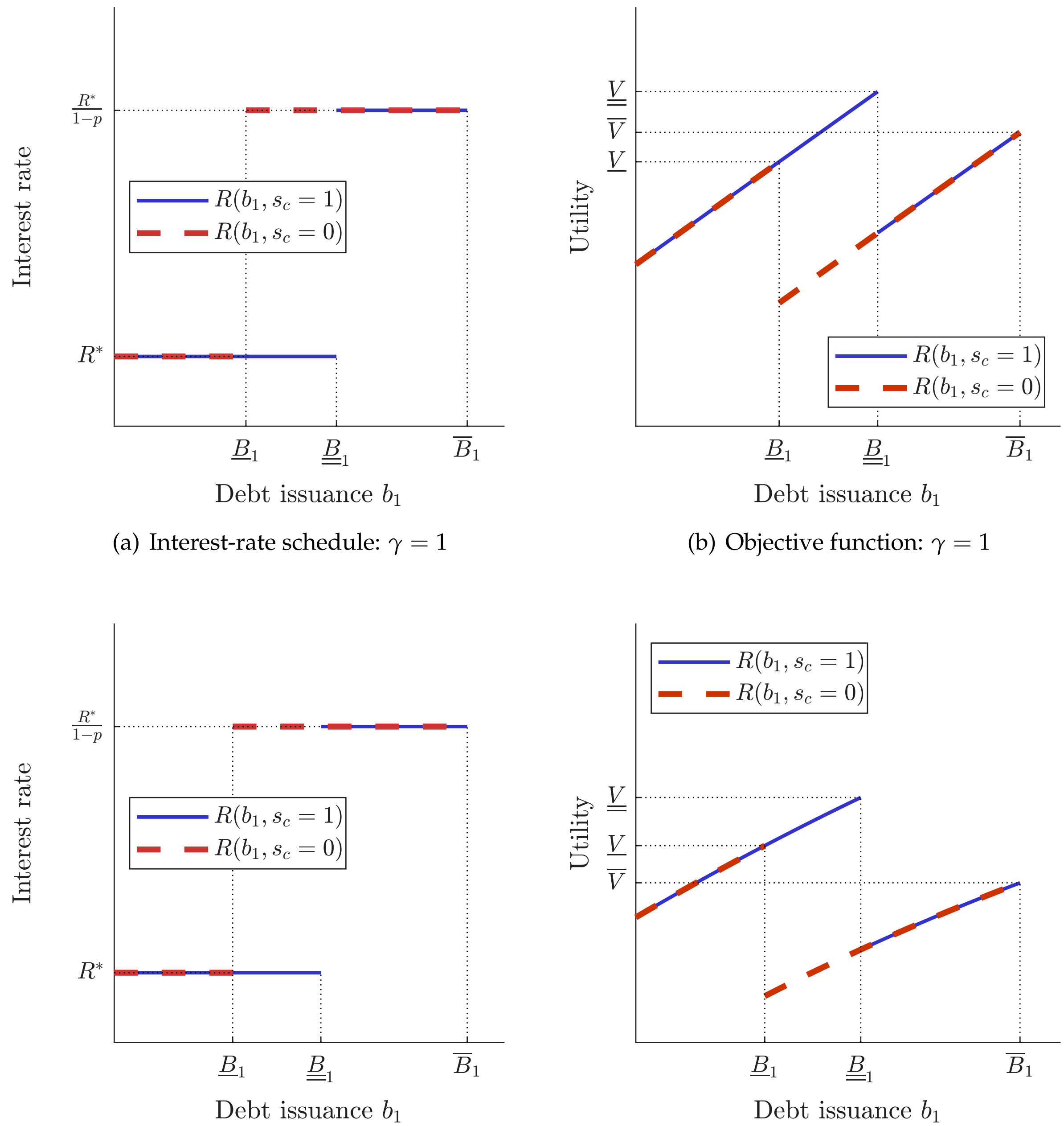

(e) Interest-rate schedule: $\beta = 0.5$




(f) Objective function: $\beta = 0.5$

Figure 9: Comparative statics with respect to the discount factor $\beta$

This result is not general, however. It arises only when the implied intertemporal reallocation of consumption remains limited, since $\gamma$ also governs the willingness to substitute consumption across time. In the infinite-horizon model analyzed in Section 3, by contrast, the effect is reversed: higher risk aversion leads the borrower to issue more debt at the higher interest rate.




(a) Interest-rate schedule: $\gamma = 1$

(b) Objective function: $\gamma = 1$

(c) Interest-rate schedule: $\gamma = 15$

(d) Objective function: $\gamma = 15$

Figure 10: Comparative statics with respect to risk aversion $\gamma$

# C Detrended Model

This appendix presents the stationary version of the infinite-horizon model described in Section 3.1. We normalize all variables by lagged income, $Y_{-1}$. Let

$$a \equiv \frac{A}{Y_{-1}}, \qquad b' \equiv \frac{B'}{Y_{-1}},$$

where $A$ denotes inherited debt service and $B'$ denotes the amount of resources raised in the current bond auction. Since $Y = gY_{-1}$, detrended income is equal to $g$. Let $s = (s^c, s^{ck})$ denote the vector of sunspot realizations.

The detrended value of repayment is

$$v^{nd}(a, g, s) = \max_{b' \leq \overline{b}(a,g,s)} \left\{ u(g - a + b') + \beta g^{1-\gamma} \sum_{g'} \sum_{s'} \Pi(g'|g) p(s'|s) v\left(a', g', s'\right) \right\},$$

where

$$a' = \frac{R(b', g, s) b'}{g}.$$

The detrended value function is

$$v(a, g, s) = \max \left\{ v^{nd}(a, g, s), v^d(g, s) \right\}.$$

The detrended value of default is

$$v^d(g, s) = u\left(g(1 - \phi)\right) + \beta g^{1-\gamma} \sum_{g'} \sum_{s'} \Pi(g'|g) p(s'|s) \left[ \theta v(0, g', s') + (1 - \theta) v^d(g', s') \right].$$

Thus, the default policy is given by

$$d(a, g, s) = \begin{cases} 1, & \text{if } v^d(g, s) > v^{nd}(a, g, s), \\ 0, & \text{otherwise.} \end{cases}$$

The borrowing limit $\overline{b}(a, g, s)$ incorporates rollover risk. When the Cole–Kehoe sunspot is bad, the borrowing limit is set to zero whenever repayment without new borrowing is worse than default:

$$u(g - a) + \beta g^{1-\gamma} \sum_{g'} \sum_{s'} \Pi(g'|g) p(s'|s) v(0, g', s') < v^d(g, s).$$

Finally, the interest-rate schedule satisfies lenders' zero-profit condition. With risk-neutral lenders, gross risk-free rate $R^*$, and the price of newly issued debt normalized to one, this condition is

$$1 = \frac{R(b', g, s)}{R^*} \sum_{g'} \sum_{s'} \Pi(g'|g) p(s'|s) \left[ 1 - d\left( \frac{R(b', g, s) b'}{g}, g', s' \right) \right].$$

# D Quantifying Implicit Transfers for Spain

In this section, we provide a quantification of the value of implicit transfers received by Spain from the European Central Bank via the Public Sector Purchase Programme (PSPP). The bond purchases started in March 2015 and have continued for the next ten years. To the extent that the ECB's holding of Spanish bonds is motivated politically, the true risk associated with these assets is likely higher than what the observed interest rate reflects. In such case, the Spanish government receives an implicit transfer by being able to pay a lower interest rate on these bonds than in the counterfactual scenario of no such intervention. Figure 11 illustrates the cumulative net purchases of Spanish government bonds over time, along with hypothetical unwinding forecasts. The purchases increased steadily in the first few years and the stock exceeded 0.25 trillion euro in 2018, falling back to this level only in 2025Q1. At the pace of the selloff that started from the peak in 2022, it would take at least another decade to unwind the entire portfolio.

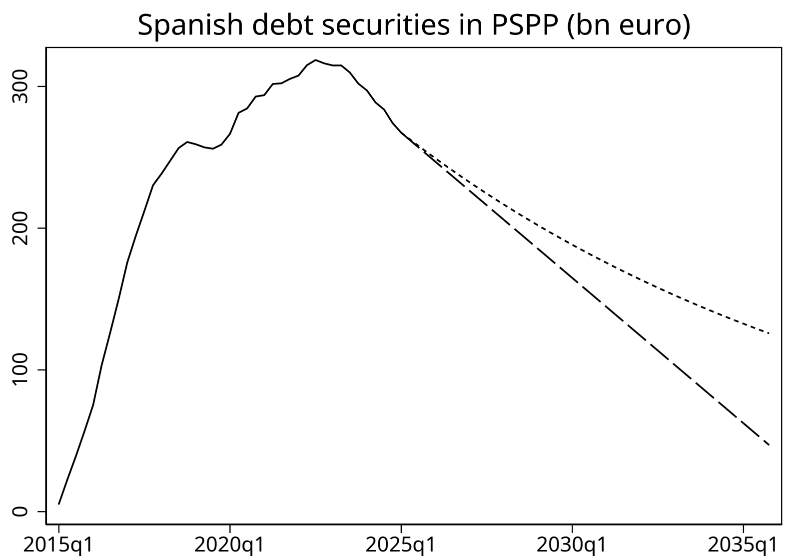



Figure 11: Cumulative net purchases for Spanish bonds under PSPP

*Data source: https://www.ecb.europa.eu/mopo/implement/app/html/index.en.html#pspp. The forecasts beyond 2025Q1 are the authors' own calculations derived by extrapolating the linear and geometric trends from the peak in 2022Q3.*

To calculate the implicit transfer, we first define a parameter $\kappa$ that captures the interest rate differential associated with the intervention. That is, assume that in the absence of the ECB's bond purchase program the true Spanish spread would have been higher by $\kappa$ percentage points. In other words,

$$\kappa \equiv \text{Yield differential absent ECB intervention}$$

We proceed as follows. We set $\kappa$ to a fixed number of percentage points. Next, for each quarter, we compute the implicit transfer arising from lower interest rates by multiplying $\kappa$ by the outstanding stock of Spanish bonds held in the ECB portfolio. Finally, we compute the present value of these transfers as of 2015Q1 (using the German government bond yield as the risk-free rate), sum them, and express the result as a share of Spanish GDP in that period.

Table 7: Implicit transfer to Spain by assumed value of $\kappa$ (% of 2015 GDP)

| **$\kappa$ (in percentage points)** | **0.1** | **0.5** | **1.0** | **1.5** | **2.0** | **2.5** |
|---|---|---|---|---|---|---|
| end sample at 2025Q1 | 0.22 | 1.11 | 2.22 | 3.33 | 4.43 | 5.54 |
| linear extrap. to 2038Q1 | 0.36 | 1.79 | 3.57 | 5.36 | 7.14 | 8.93 |
| geometric extrap. to 2038Q1 | 0.40 | 1.98 | 3.96 | 5.94 | 7.92 | 9.90 |

*Note: in extrapolating series, we assume the risk-free interest rate in all periods beyond 2025Q1 is the average rate over 2022Q3-2025Q1 equal to 2.27%.*

Table 7 presents the results of our calculation for different values of $\kappa$, and for different assumptions about the unwinding of the ECB-held bonds beyond the latest data observation (2025Q1). When we only consider the actually observed data (that is, we assume that ECB sells all Spanish bonds in 2025Q1), the total implicit transfer varies between 0.2% and 5.5% of Spain's GDP in 2015 for $\kappa$ ranging between 0.1 and 2.5 percentage points. To see that this is a reasonable interval, note that average spread on Spanish bonds observed between 2015Q1 and 2025Q1 was 1 percentage point, while the maximum spread topped at 5.1 percentage points in 2012Q3.[24] On the other hand, if we extrapolate the bond holdings into the future (given that in 2025Q1 the ECB still held over 0.25 trillion euro worth of Spanish bonds as Figure 11 shows), the total value of the implicit transfer ranges from 0.4% to around 9-10% of 2015 GDP depending on the extrapolation method.

Naturally, the calculation above is designed to illustrate magnitudes and should be taken with a grain of salt. On the one hand, we derive the implicit transfer only from the bonds actually held by the ECB and ignore any spillover on the bonds held by private investors (by increasing confidence). On the other hand, large asset purchases may also have generated moral hazard on the part of the Spanish government, inducing it to issue excessive debt (especially during the Covid-19 episode). Ultimately, we ignore these hard to quantify forces that work in opposite directions and focus on what we can observe directly.

[24] Altavilla et al. (2016) estimate that OMT announcements reduced the yield on Spanish two-year government bond by 2 percentage points, which is well within the interval we consider in Table 7.